GNSS (GPS and GLONASS) Positioning
used in
Land Surveying and Engineering

by

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on
GPS Best Practices Workshop
Winnipeg, Manitoba, Canada
January 18, 2011

- when it has to be right

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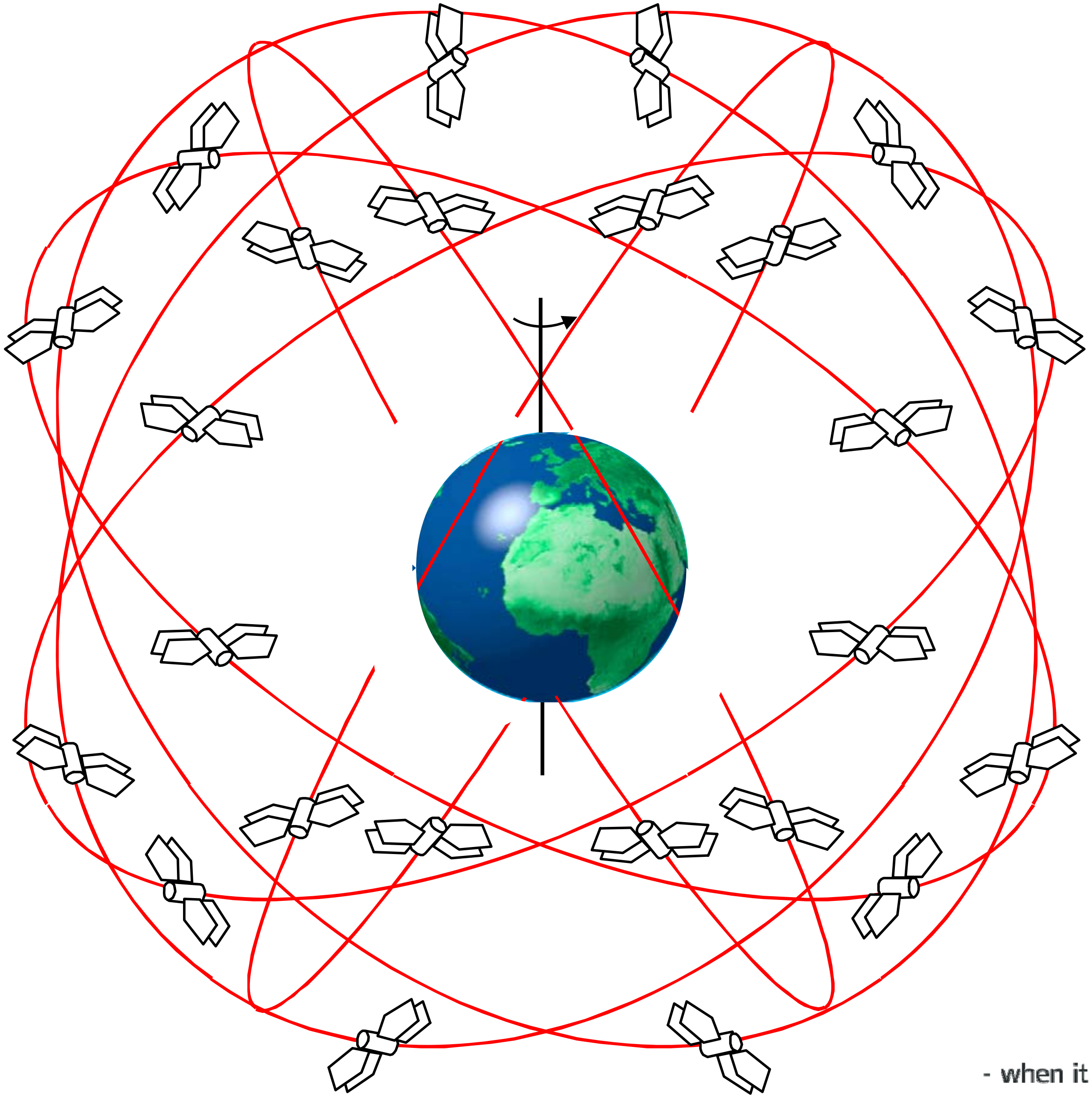
Presentation Outline

Contents:

- ***Review and Current Status of GPS and GLONASS Satellite Systems***
- ***GNSS Measurement and Positioning Processes***
- ***Real-Time GNSS Positioning Methods***
- ***GNSS Positioning Reliability Assessment***
- ***Transformation of GNSS Positions to Local Coordinate Systems***
- ***Summary and Conclusion***

Review and Current Status of GPS and GLONASS Satellite Navigation Systems

General GNSS Constellation Layout



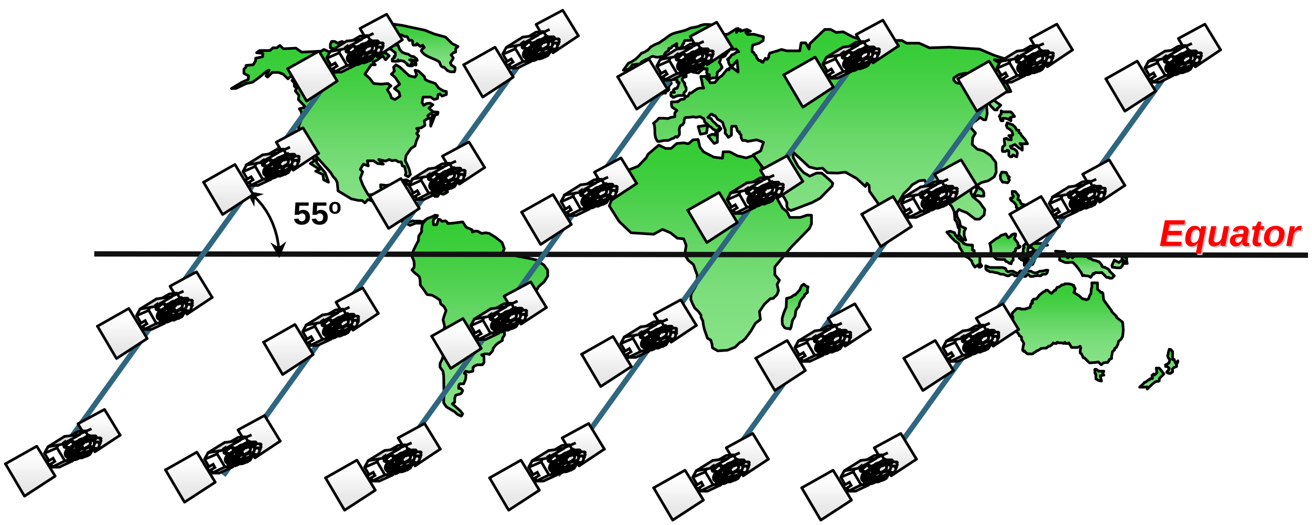
GPS Constellation Characteristics

32 Satellites (24 + 8 spare-extra)

- **6 orbital planes inclined at 55º**

  (4 Satellites + spares per orbital plane)
- **21,000 km above the Earth**
- **12-hour Orbital Period**
- **Transmitting Code and Phase Data on**

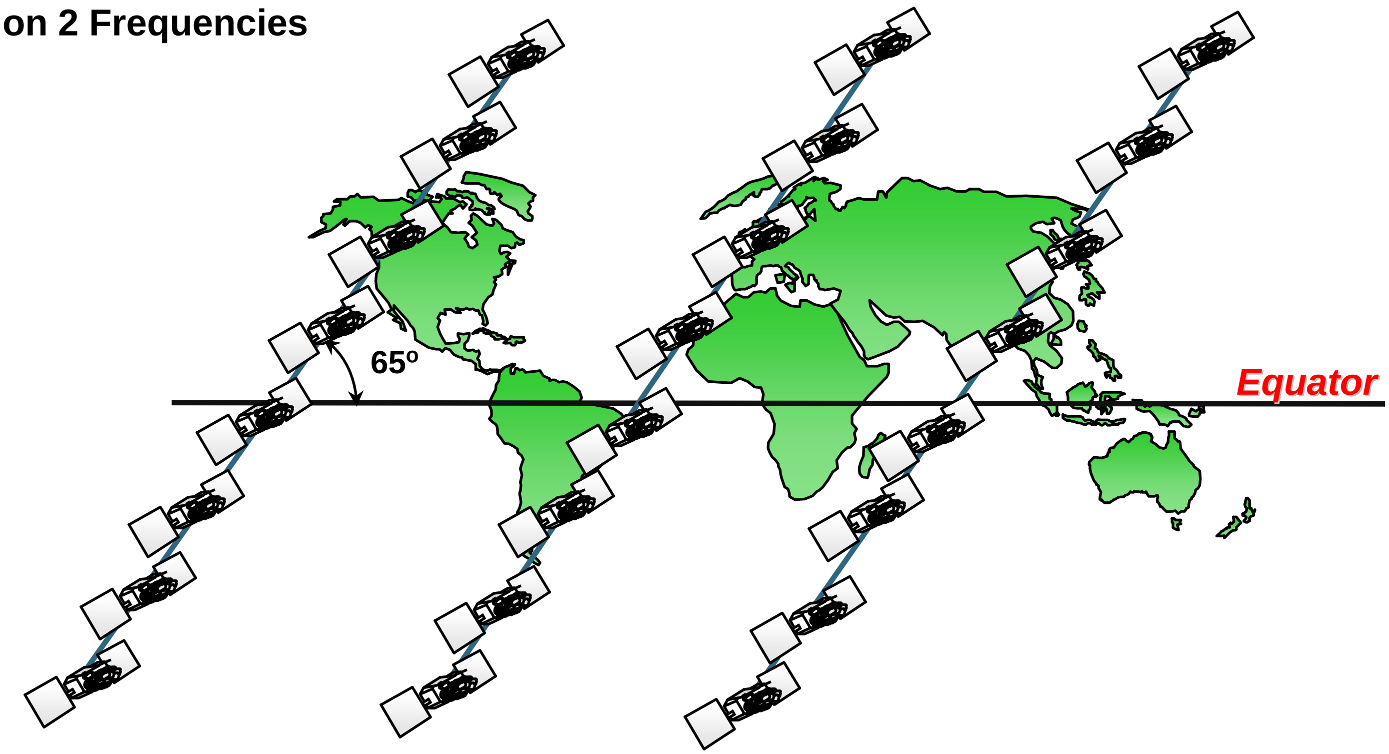
  2 Frequencies



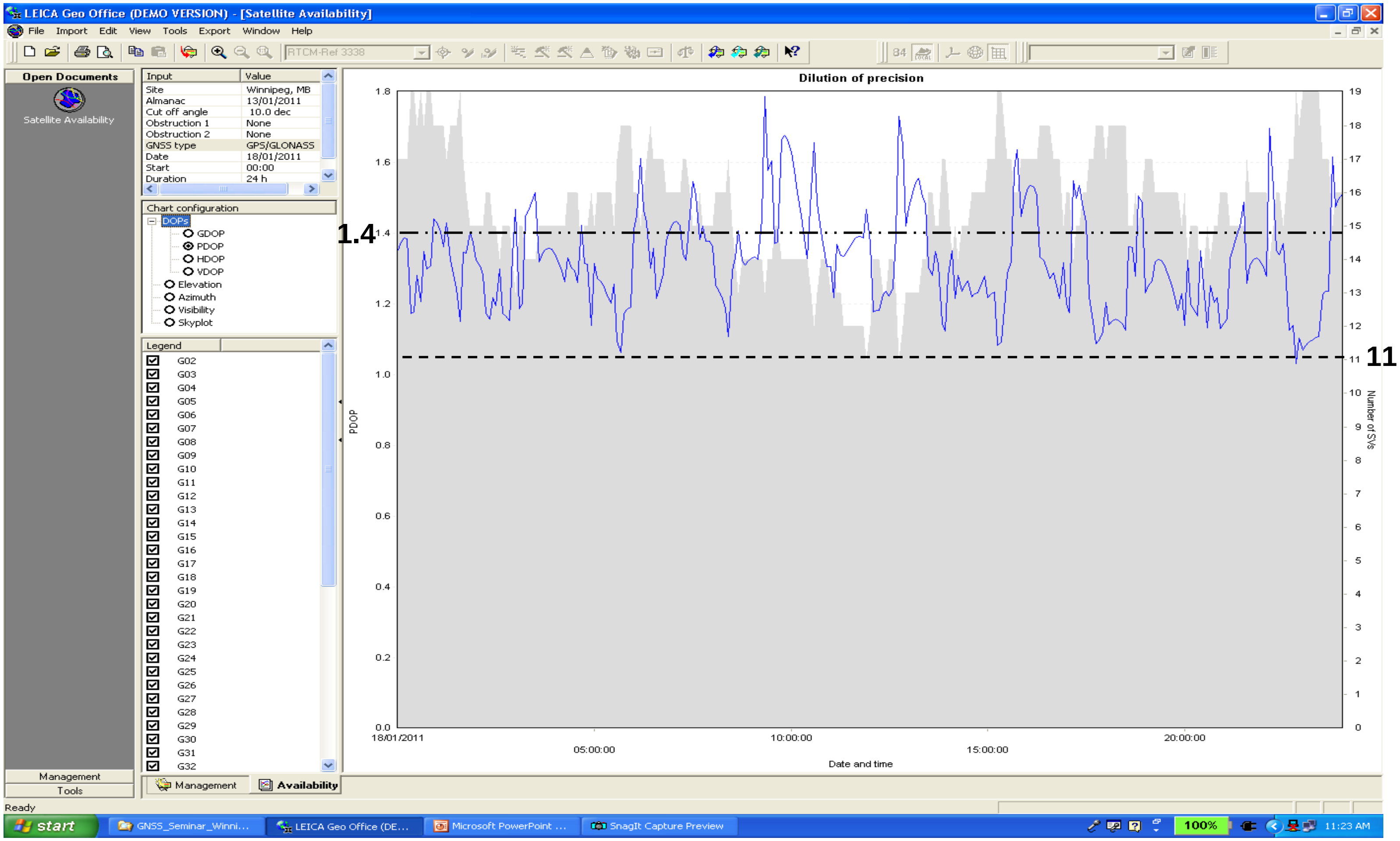
GLONASS Constellation Characteristics

24 Satellites

- **3 orbital planes inclined at 65º**
  (8 Satellites per orbital plane)
- **19,000 km above the Earth**
- **11-hour Orbital Period**
- **Transmitting Code and Phase Data**
  on 2 Frequencies



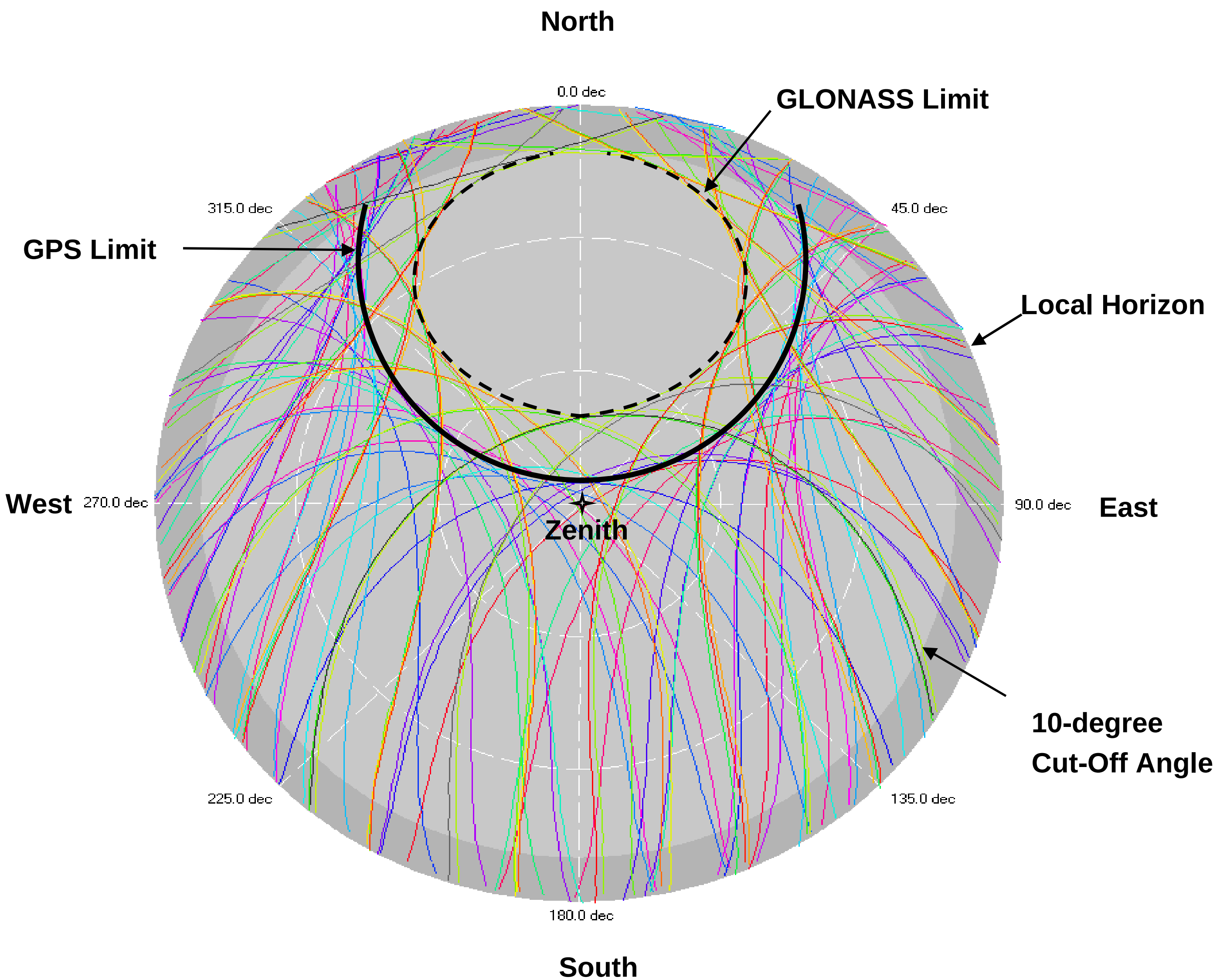
GNSS Satellite Availability for Winnipeg January 18, 2011



Date	13/01/2011
GNSS Type	GPS/GLONASS
Number of Satellites	52
Satellites GPS	G02, G03, G04, G05, G06, G07, G08, G09, G10, G11, G12, G13, G14, G15, G16, G17, G18, G19, G20, G21, G22, G23, G24, G25, G26, G27, G28, G29, G30, G31, G32
Satellites GLONASS	R01, R02, R05, R06, R07, R08, R09, R10, R11, R12, R13, R14, R15, R16, R18, R19, R20, R21, R22, R23, R24

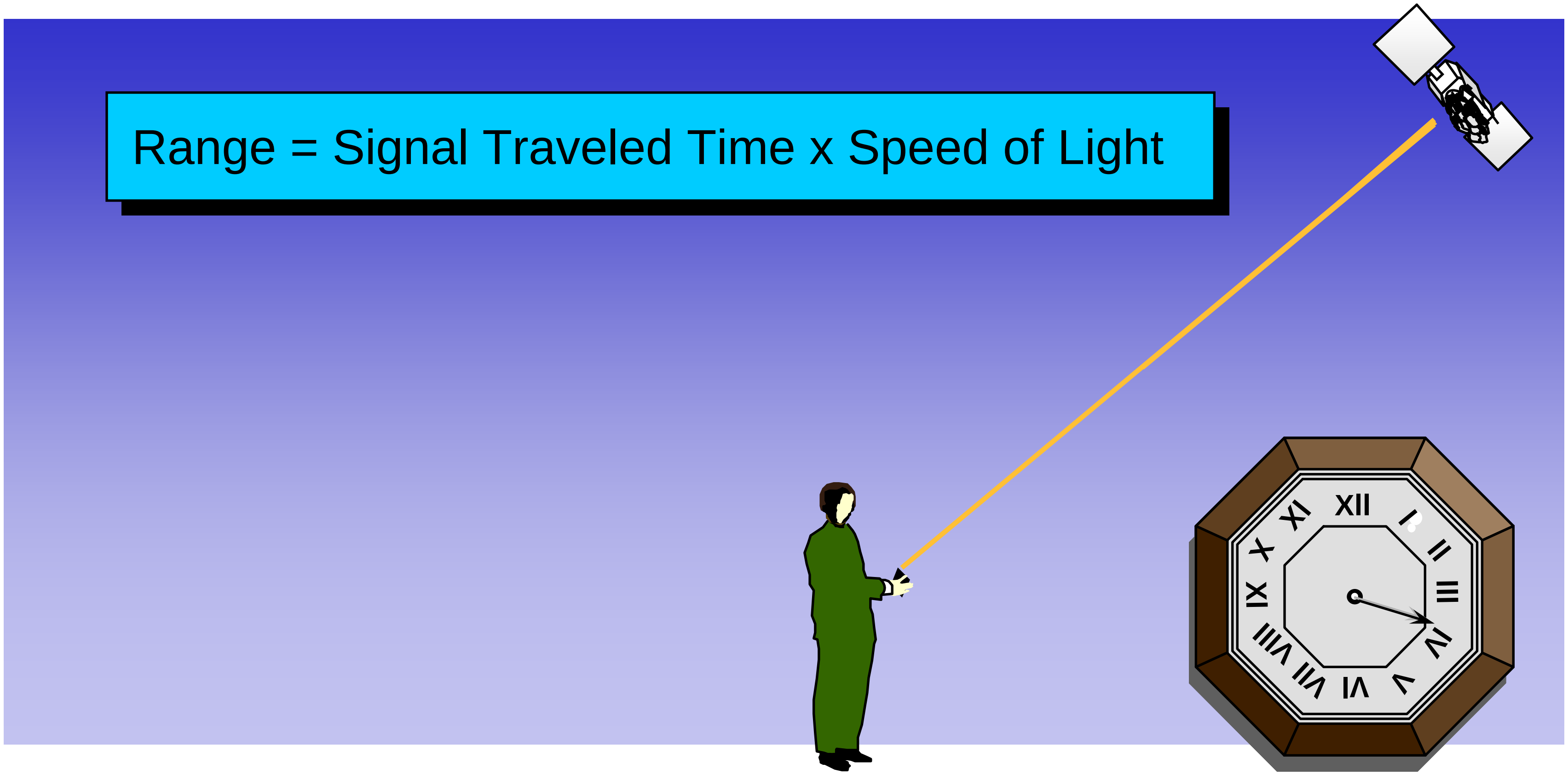
31 GPS and 21 GLONASS for 11 or more visible Satellites with an Average PDOP of 1.4

GNSS Satellite Sky Plot for Mid-Northern Latitudes



GNSS
Measurement and Positioning Processes

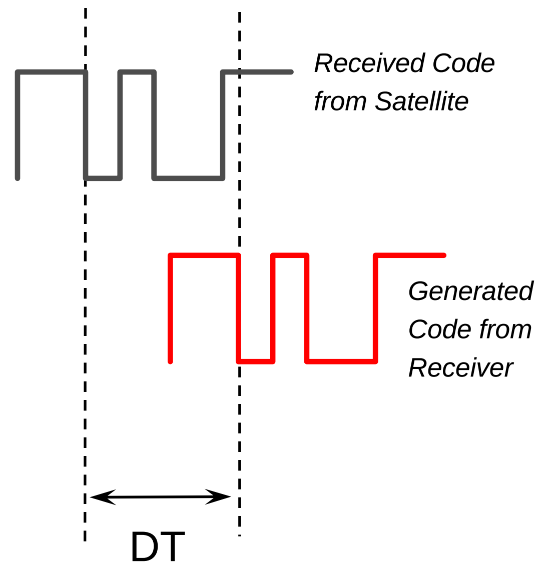
GNSS Range Measurement Principle



Range Determination from Code Measurements

Code Observations

- Each satellite sends a coded signal which repeats itself regularly
- Receiver compares self generated code signal from a set of known library codes with te received code signal
- From the time difference **DT**, a range observation can be determined by multiplying it by the Speed of Light **c**
- Receiver clock needs to be synchronized with the satellite clock
- Code Measurements are complete and robust but are not too precise (+/- 30 cm) when compared with Phase Measurements

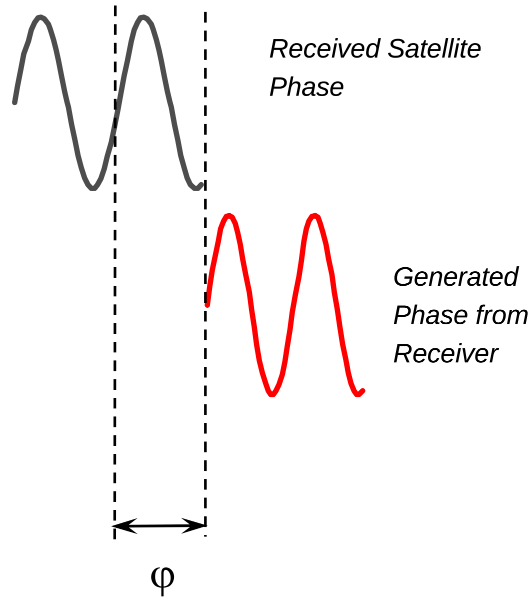


$$D = c\,(DT)$$

Range Determination from Phase Measurements

Phase Observations

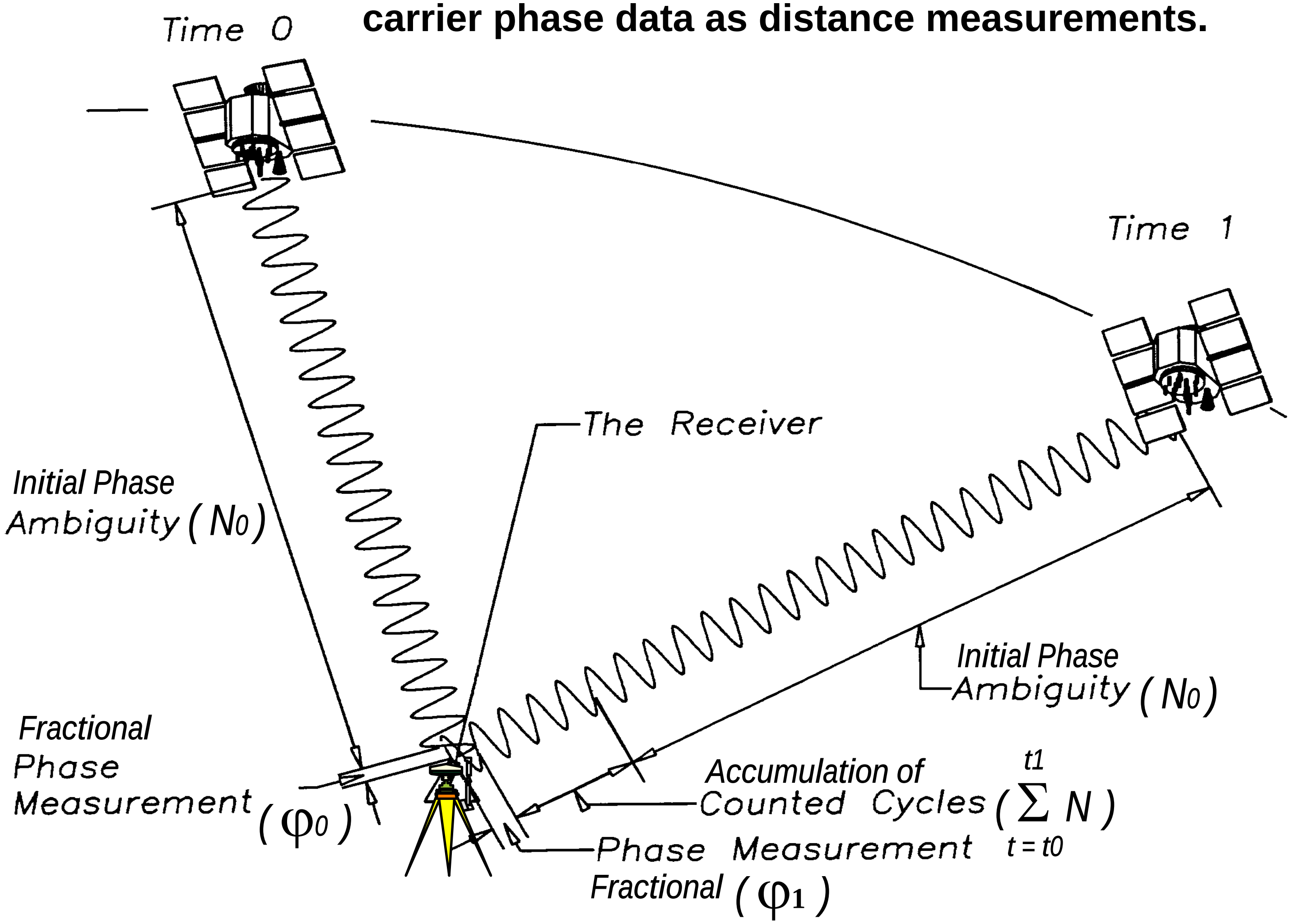
- Wavelength λ of the signal is about 20 cm
- Receiver compares self-generated phase with received phase φ
- The Total number of wavelengths is not initially known at the time the receiver is switched on (initial carrier phase ambiguity)
- As long as the satellite is continuously tracked, the change in distance can be observed and the initial carrier phase ambiguity remains constant
- Phase Measurements are very precise (+/-1 mm) but are initially not accurate until Initial Phase Integer Values **N** are resolved.
- Phase Measurements must remain un-interrupted



$$D = \varphi\lambda + N\lambda$$

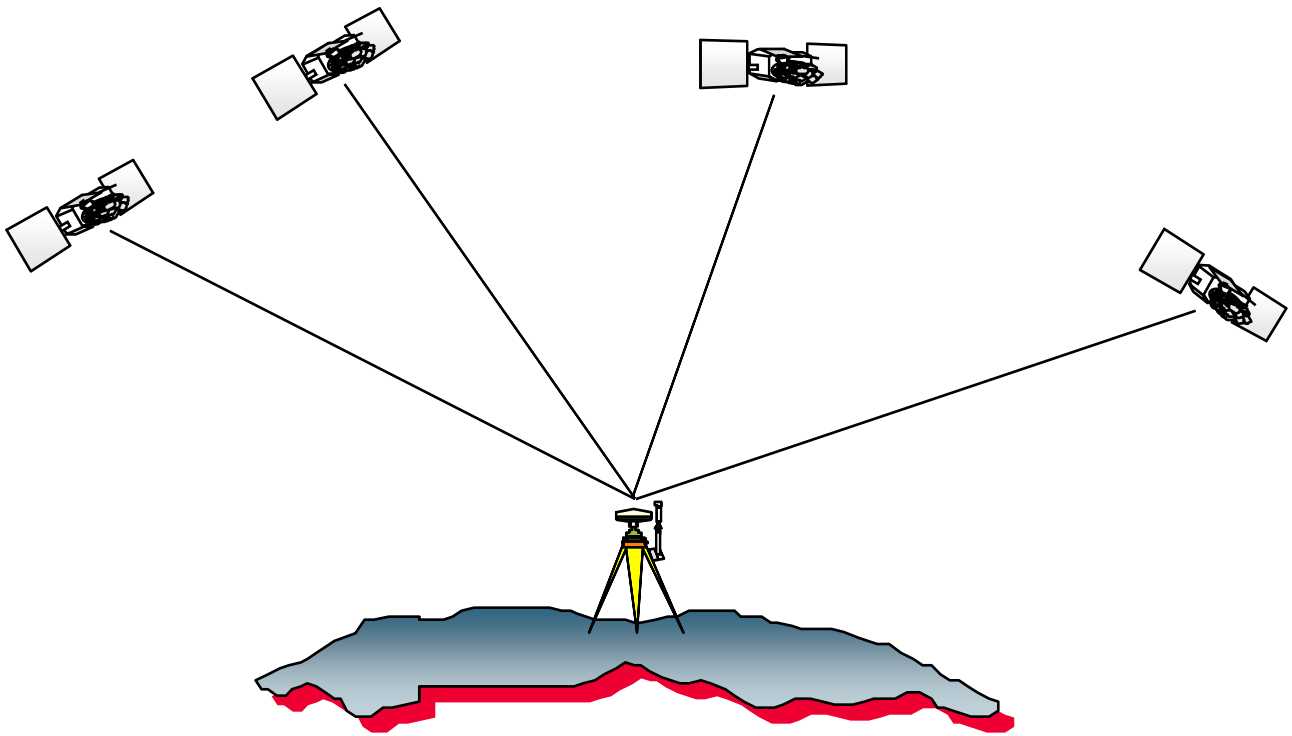
Range Observations from Accumulated Phase Measurements

Initial phase ambiguity must be determined to use carrier phase data as distance measurements.



$$D_{ti} = \phi_{ti} . \lambda = (\mathbf{N_0} + \sum_{t=t0}^{ti} N + \varphi_{ti}) . \lambda$$

(Absolute) Point Positioning (Spatial Trilateration)



Instantaneous Point Positioning from Code Measurements using Broadcast Orbits :

+/- 1 to 2 m Horizontally
+/- 3 to 5 m Vertically

Post-Processed Point Positioning from Precise Orbits and Phase Measurements accumulated after 6 or more hours:

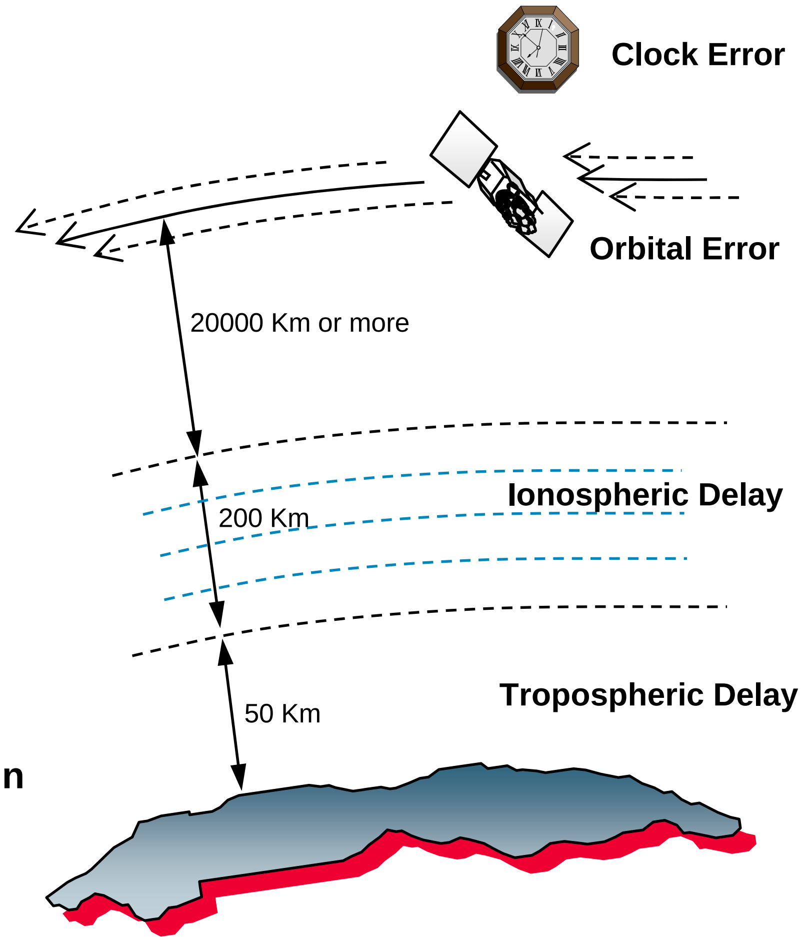
+/- 1 to 2 cm Horizontally
+/- 3 to 5 cm Vertically

GNSS Satellite Observation Errors

- **Broadcast Orbit Uncertainty (3 to 5 metres) :**
  GNSS satellite positions are not perfectly known in space from the Broadcast Ephemerides. They drift slightly from their precise trajectories.

- **Satellite Clock Error (1.0 x10**-11 sec) :**
  Timing is critical to GNSS. Even though they use atomic clocks, they are still subject to small inaccuracies in their time keeping
  These inaccuracies will translate into positional errors.

- **Atmospheric Delays (10 to 50 cm):**
  Ionospheric Layer depends on the Density and Stability of the Total Electron Content contained in the higher atmosphere.
  Tropospheric Layer depend on the Intensity and Stability of Temperature, Pressure and Humidity contained in the lower atmosphere.
  Ionospheric and Tropospheric Layers produce delays in GNSS Measurements.



(Relative) Differential Positioning (Baseline Vector)

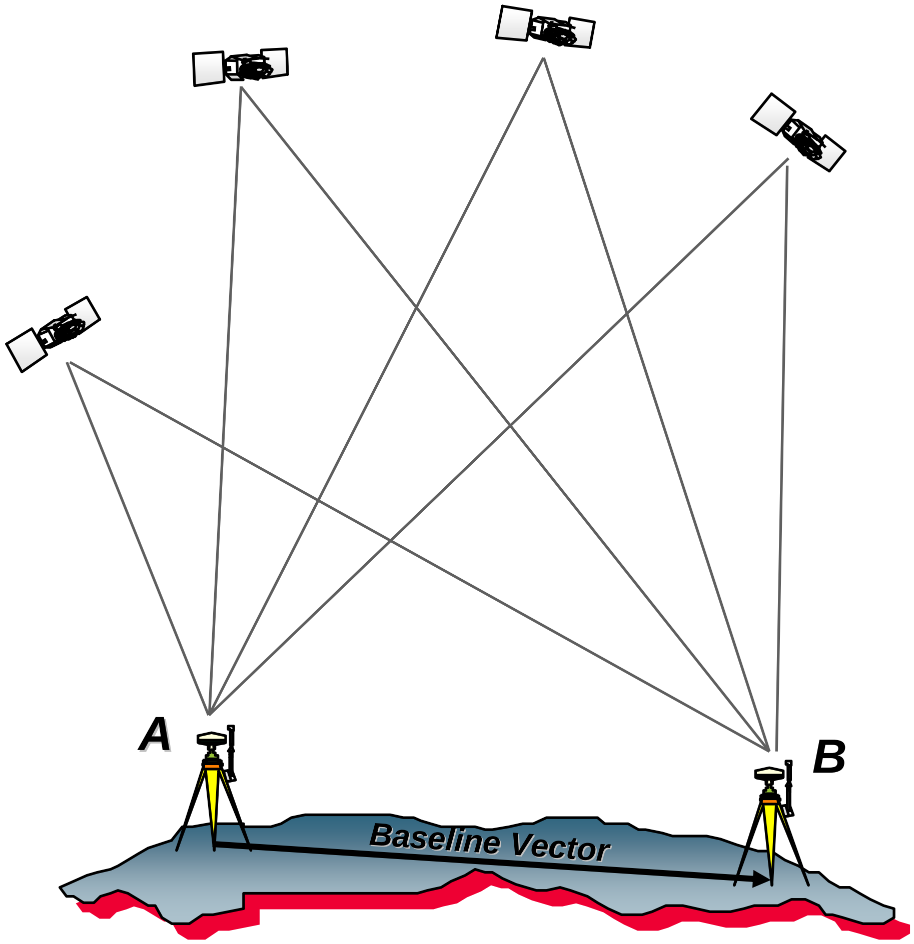
- Differential Positioning
  - Eliminates errors in satellite orbits and clocks
  - Minimizes atmospheric delays
  - Provides Relative Positioning between 2 Receivers simultaneously tracking the same Satellites.
  - For Phase Measurements:

    +/- (5 mm + 0.5 PPM) Horizontal

    +/- (10 mm + 0.5 PPM) Vertical
  - For Code Measurements:

    +/- (0.5 m + 1 PPM) Horizontal

    +/- (1.0 m + 1 PPM) Vertical

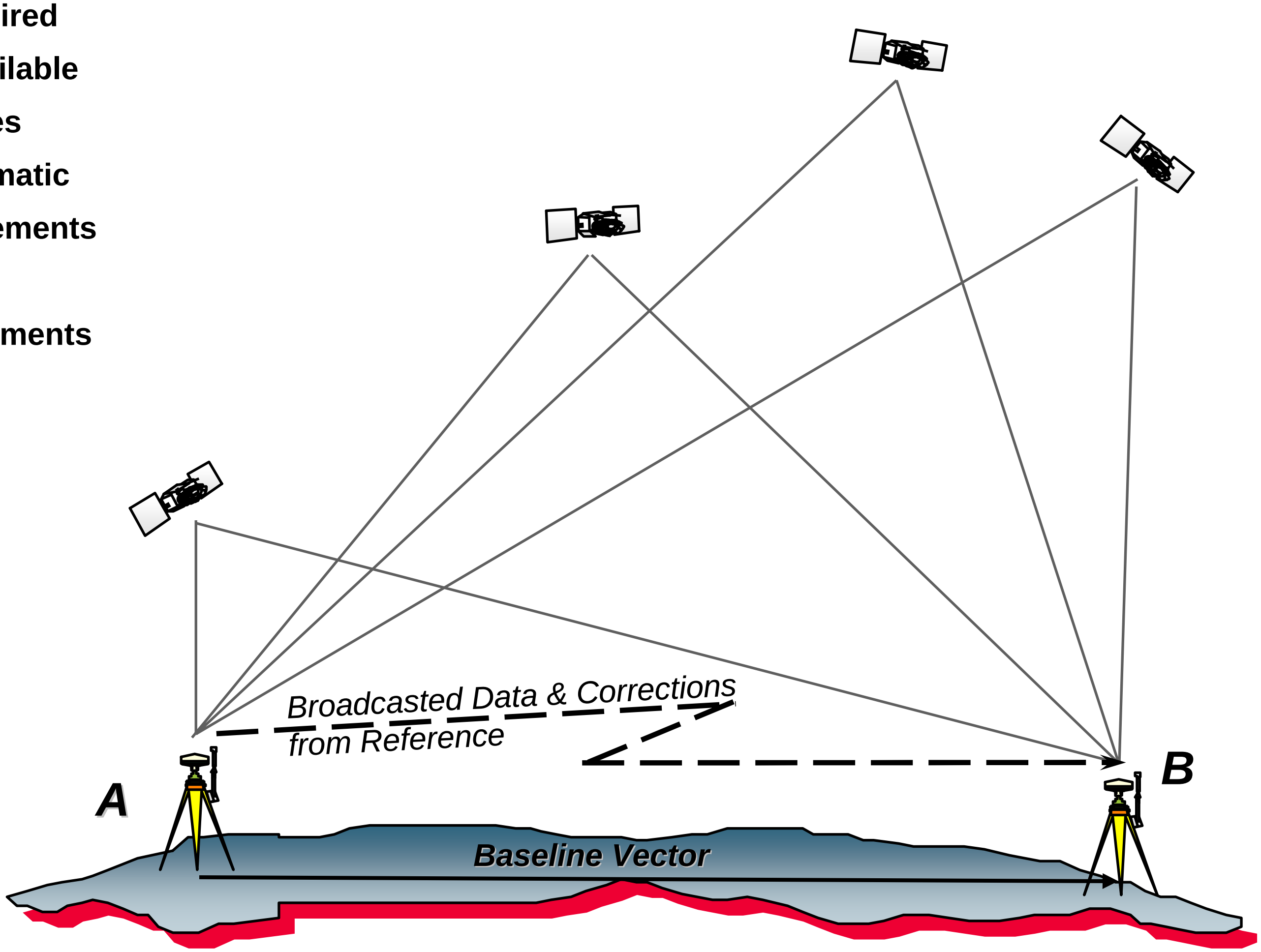


Real-Time GNSS Positioning Methods

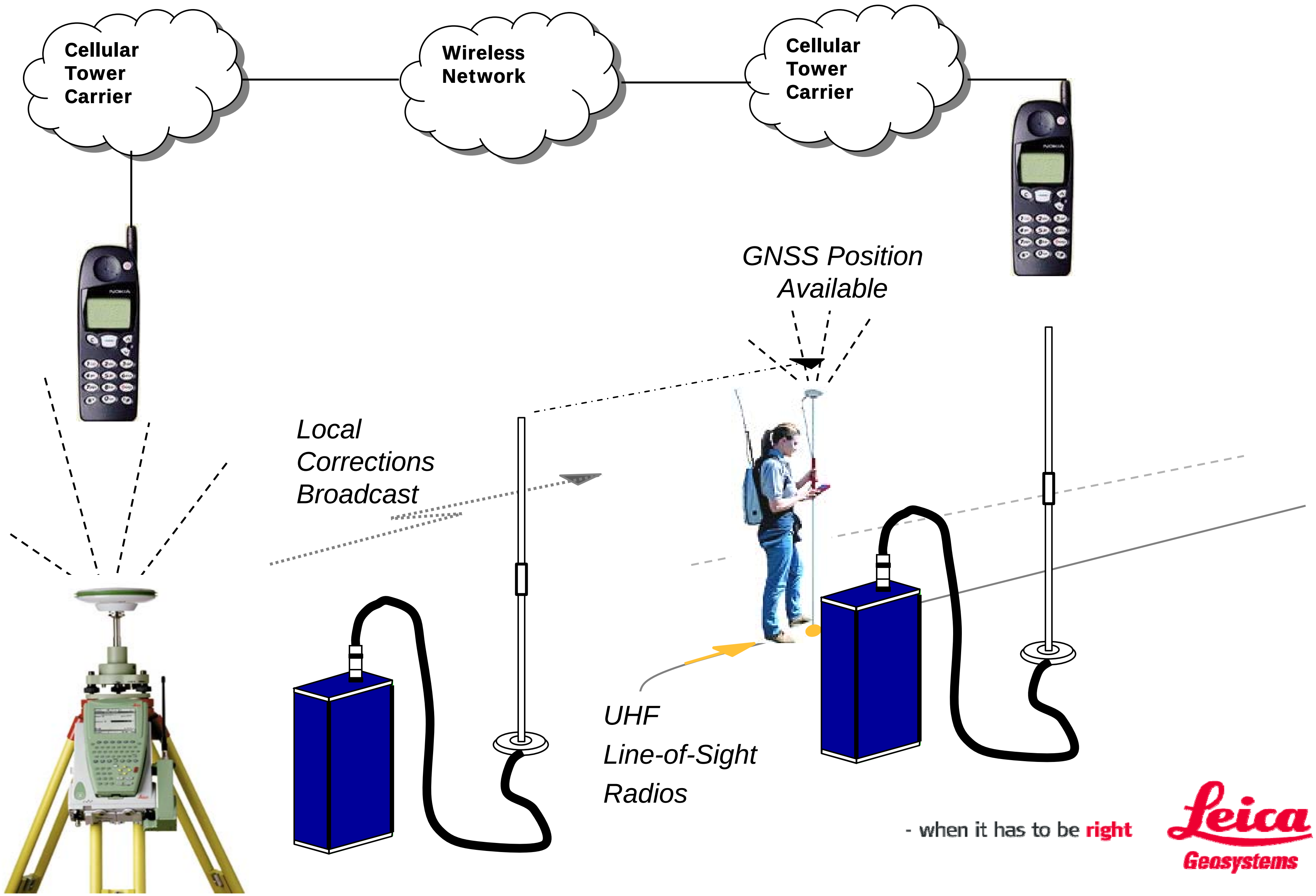
Real-Time GNSS Differential Positioning Principle

Real Time Code and Phase Positioning

- **No post processing required**
- **Results are instantly available**
- **Can operate in two modes**
  - **RTK Real-Time Kinematic**
    using Phase Measurements
  - **RT-DGPS**
    using Code Measurements



Single Base Real-Time GNSS Positioning Method

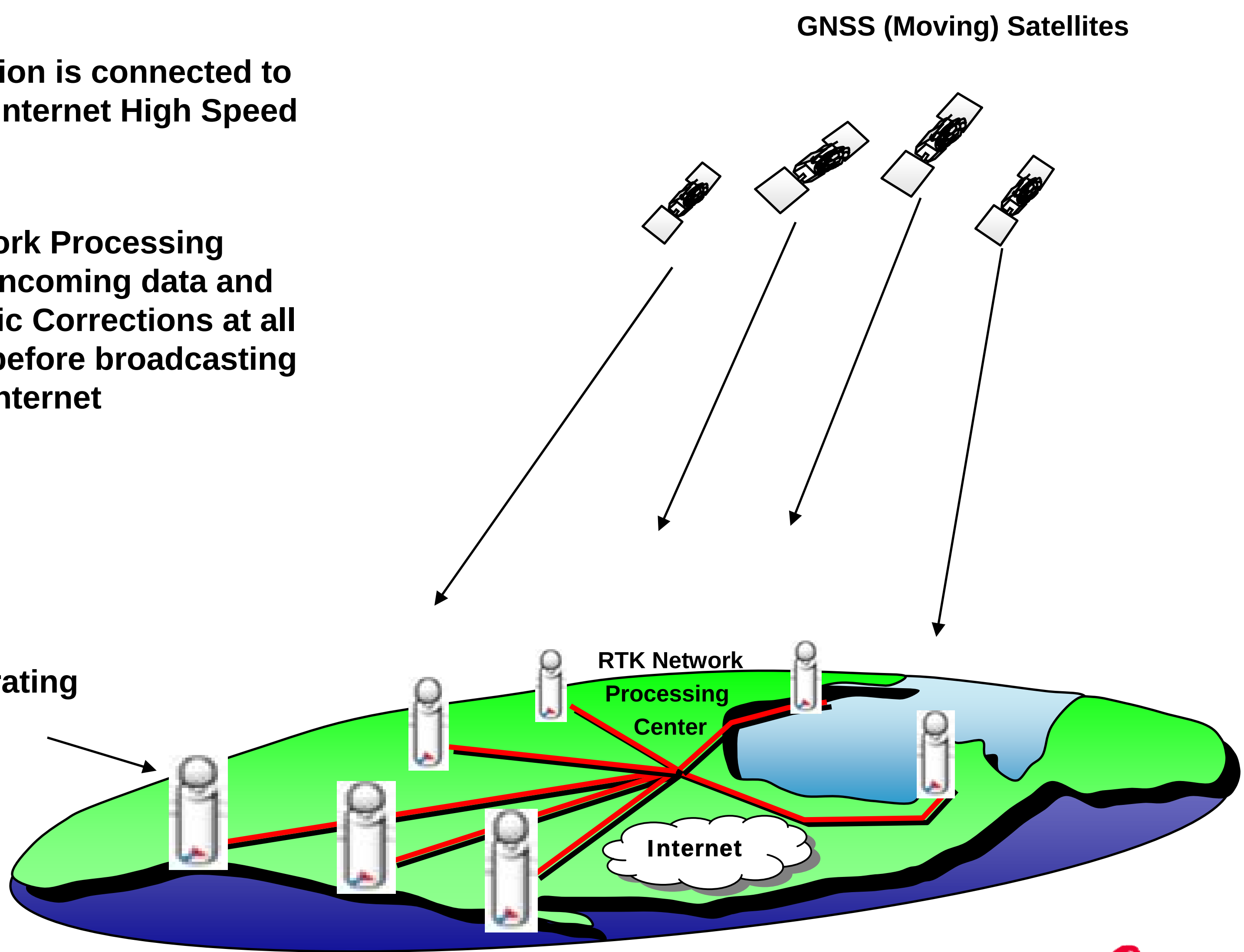


Multi-Base RTK Reference Station Network Method

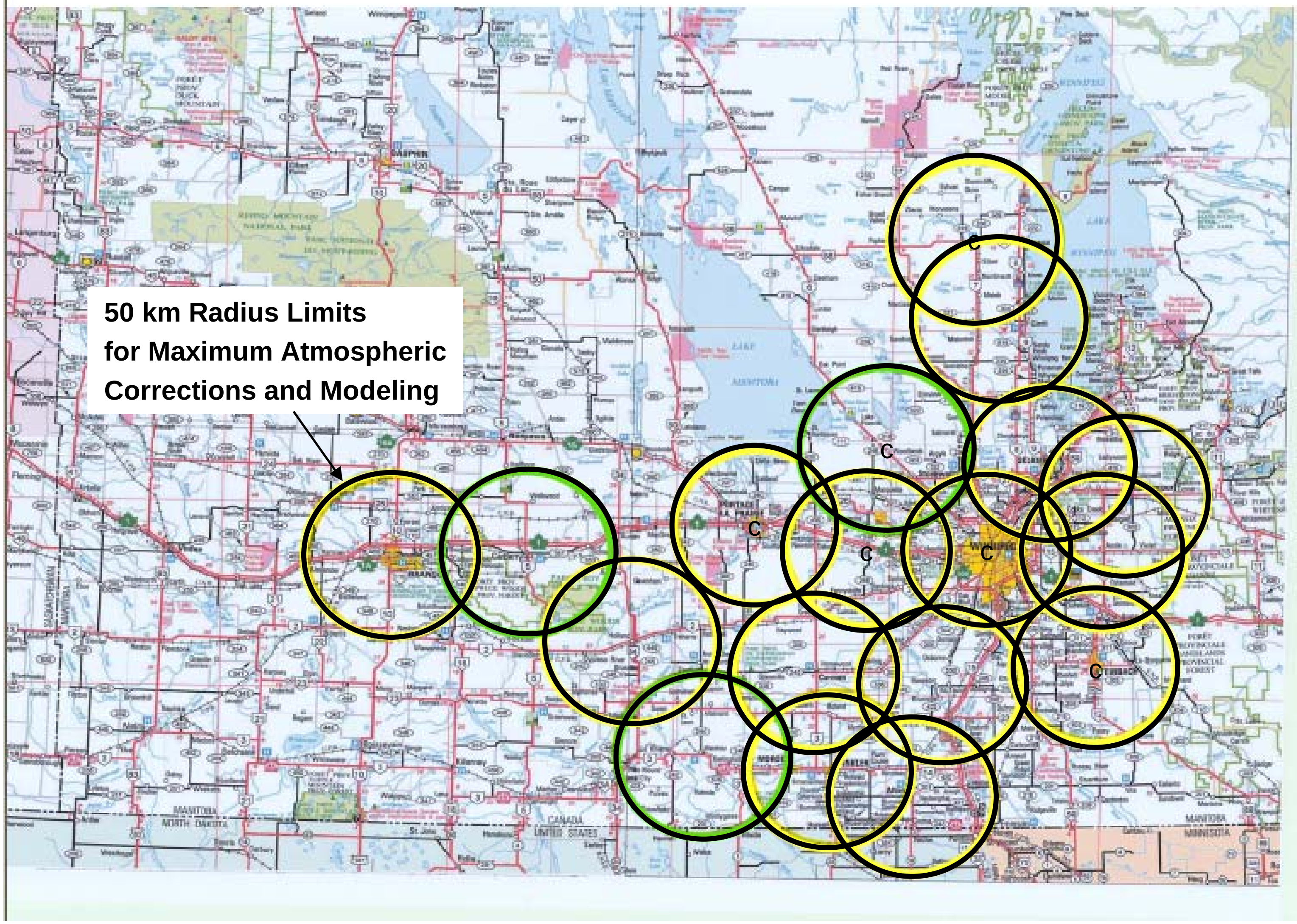
Each Reference Station is connected to a Central server via Internet High Speed line

A Central RTK Network Processing Centre validates all incoming data and estimate Atmospheric Corrections at all Reference Stations before broadcasting information via the Internet

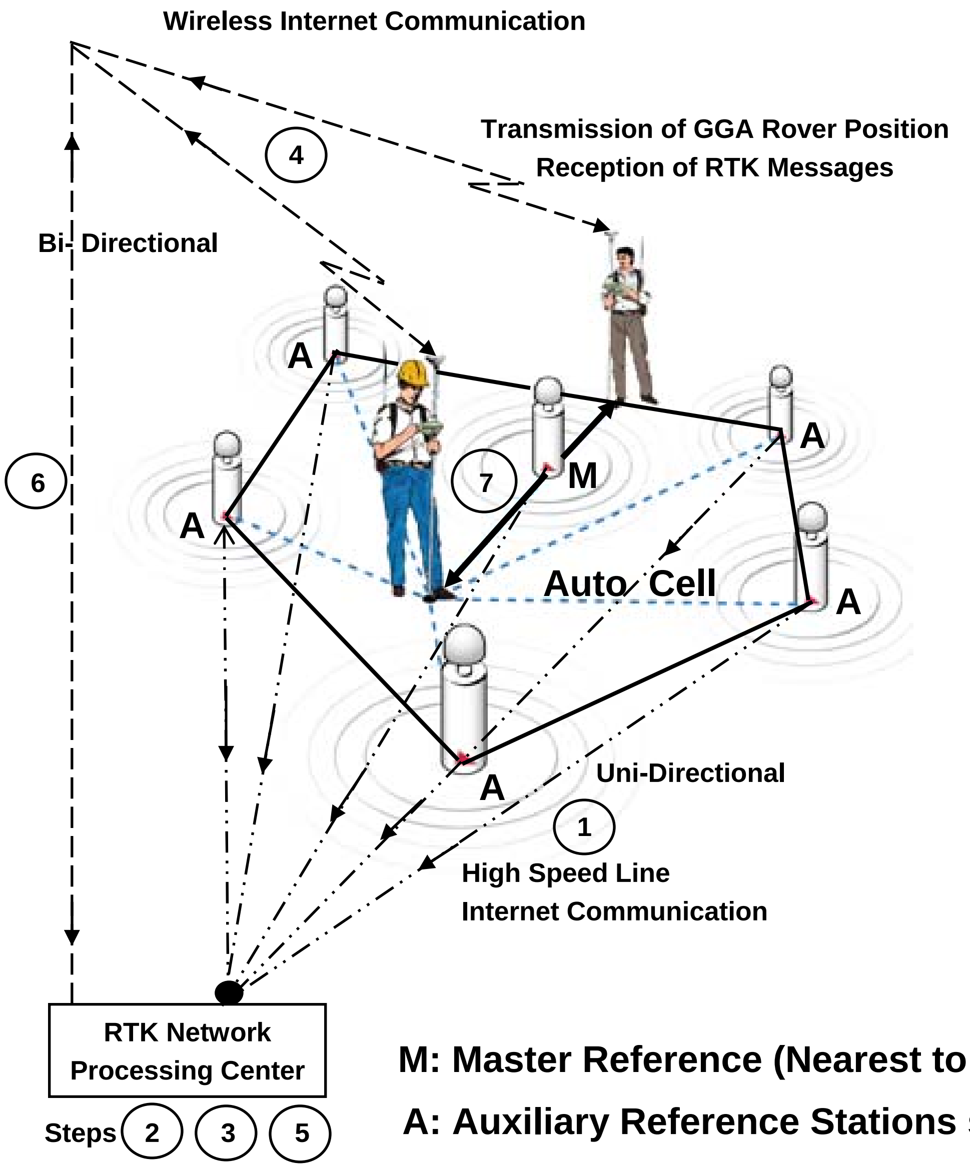
Continuously Operating Reference Station



RTK Reference Station Network in Southern Manitoba



RTK Network Positioning Data Flow



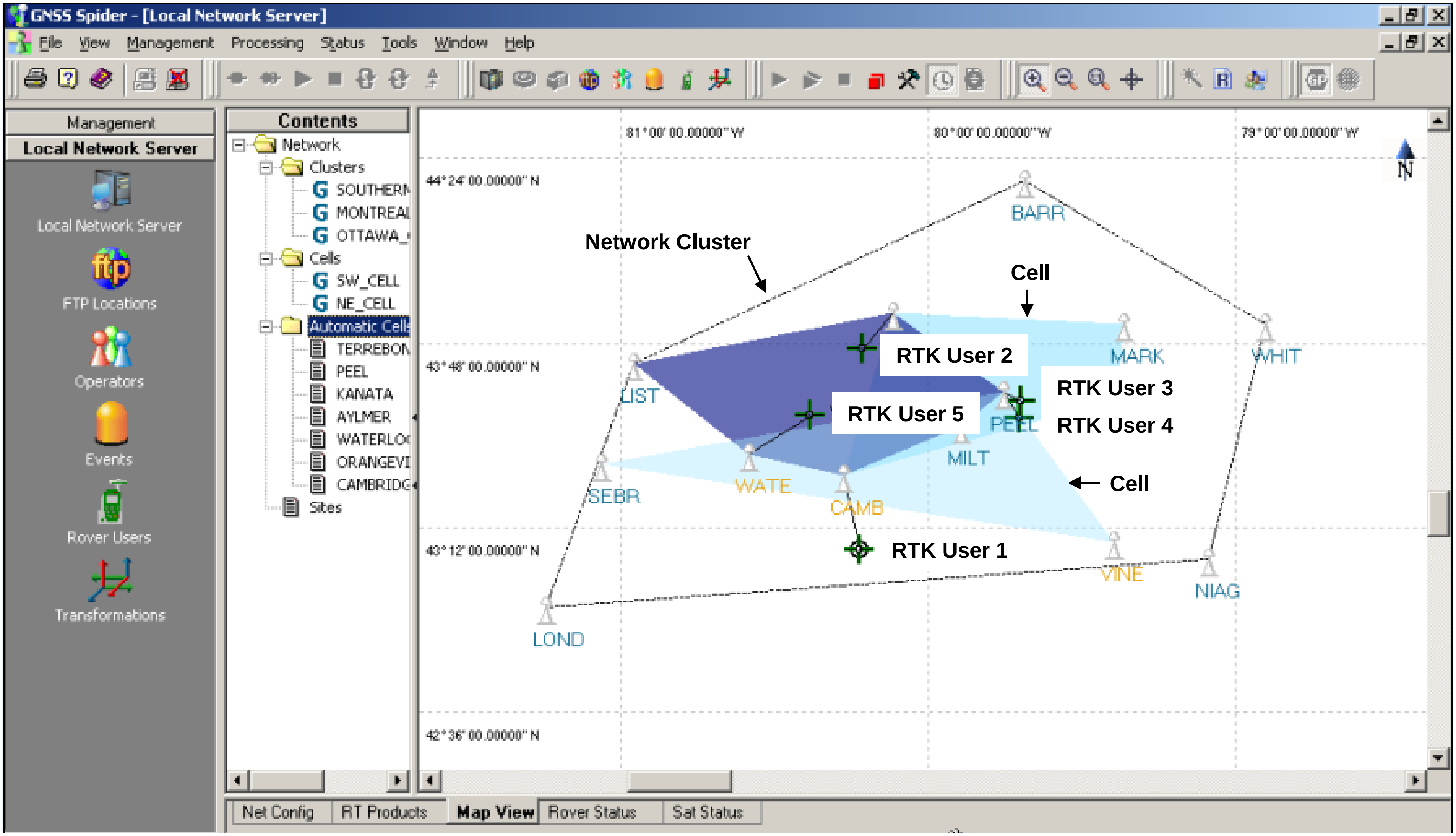
Data Processing Steps

1. Reference Station Data Gathering
2. Common Level Ambiguity Determination and Clock Correction from Cluster Stations
3. Ionospheric and Troposheric Corrections to Single Difference Phase Corrections
4. Connection Authentication and Selection of RTK Message Types
5. Assignment of Nearest (Master) and Surrounding (Auxiliary) Reference Stations in Auto Cell
6. Transmission of RTK Network RTCM V3.1 (MAC) Messages
7. RTK Network Rover Positioning in the Field from Nearest Reference Station in Auto Cell

M: Master Reference (Nearest to Rover) Station in Auto Cell

A: Auxiliary Reference Stations surrounding Rover in Auto Cell

Multi-User Network RTK Solutions



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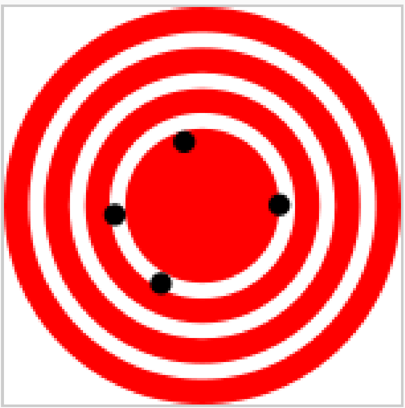
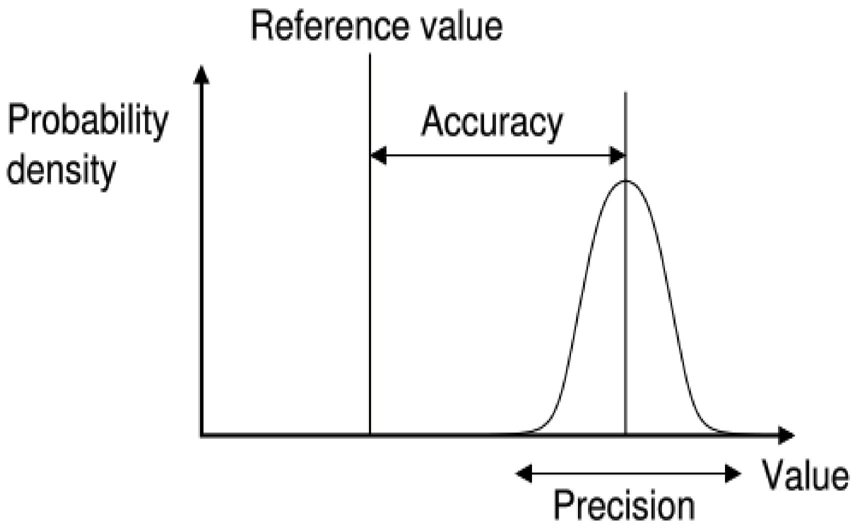
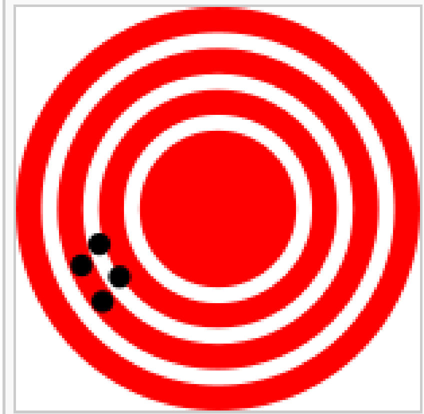
GNSS Positioning Reliability Assessment

GNSS Coordinate Quality (CQ) Values



High accuracy, but low precision

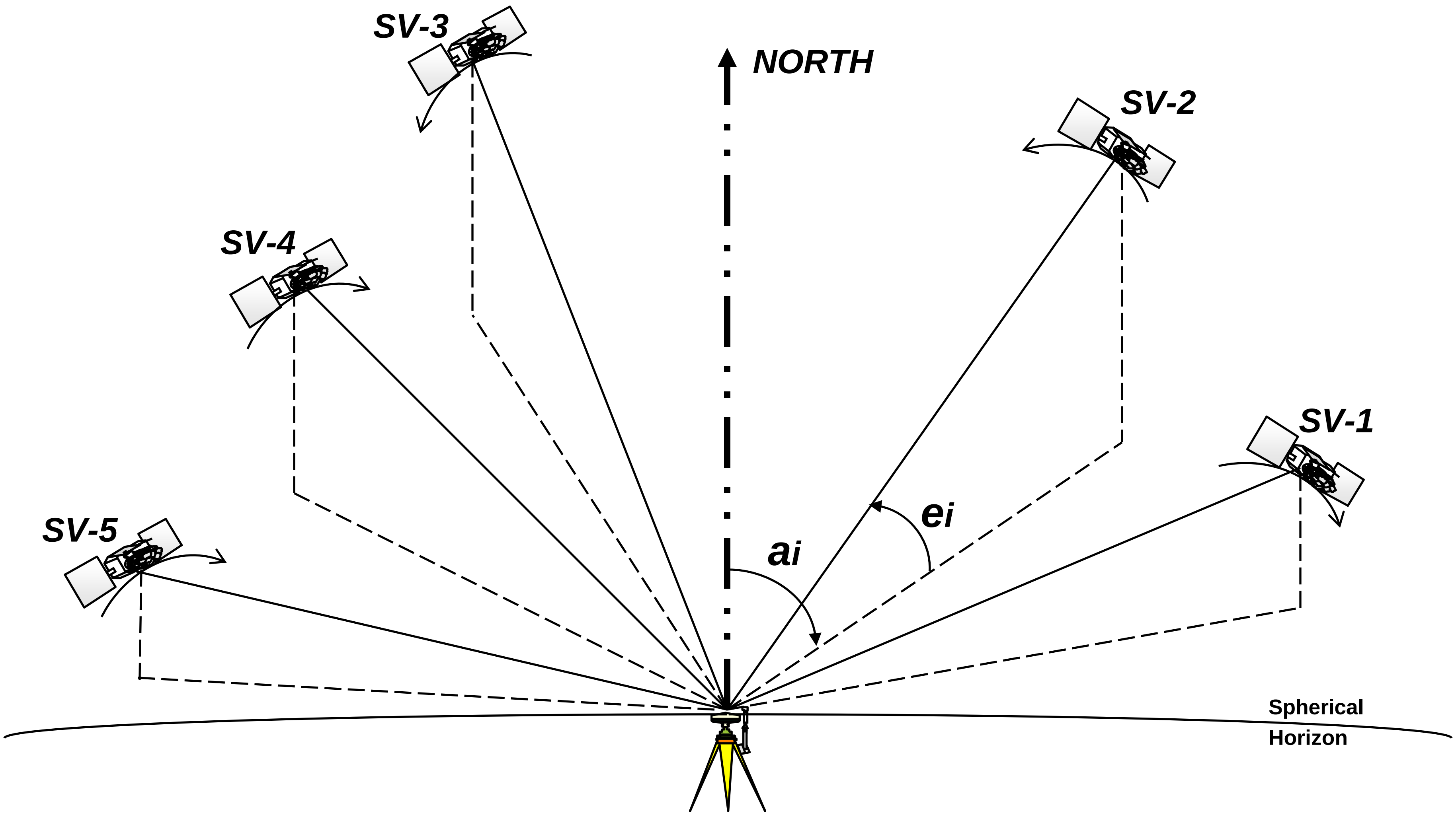




High precision, but low accuracy

What are Coordinate Quality Values?
How are they derived?
Which phenomena affect them?
How to properly interpret them?

GNSS Satellite Positioning Geometry



Dilution Of Precision (DOP) from Satellite Geometry

DOP Matrix

Dilution Of Precision $=$

$$\begin{bmatrix} ndop^2, & nedop, & nvdop, & ntdop \\ & , edop^2, & evdop, & etdop \\ & & , vdop^2, & vtdop \\ & & & , tdop^2 \end{bmatrix}$$

where:

$GDOP = (+/-)\ (ndop^2 + edop^2 + vdop^2 + tdop^2)^{1/2}$ ***Geometric DOP***

$PDOP = (+/-)\ (ndop^2 + edop^2 + vdop^2)^{1/2}$ ***3-D Position DOP***

$HDOP = (+/-)\ (ndop^2 + edop^2)^{1/2}$ ***2-D Horizontal DOP***

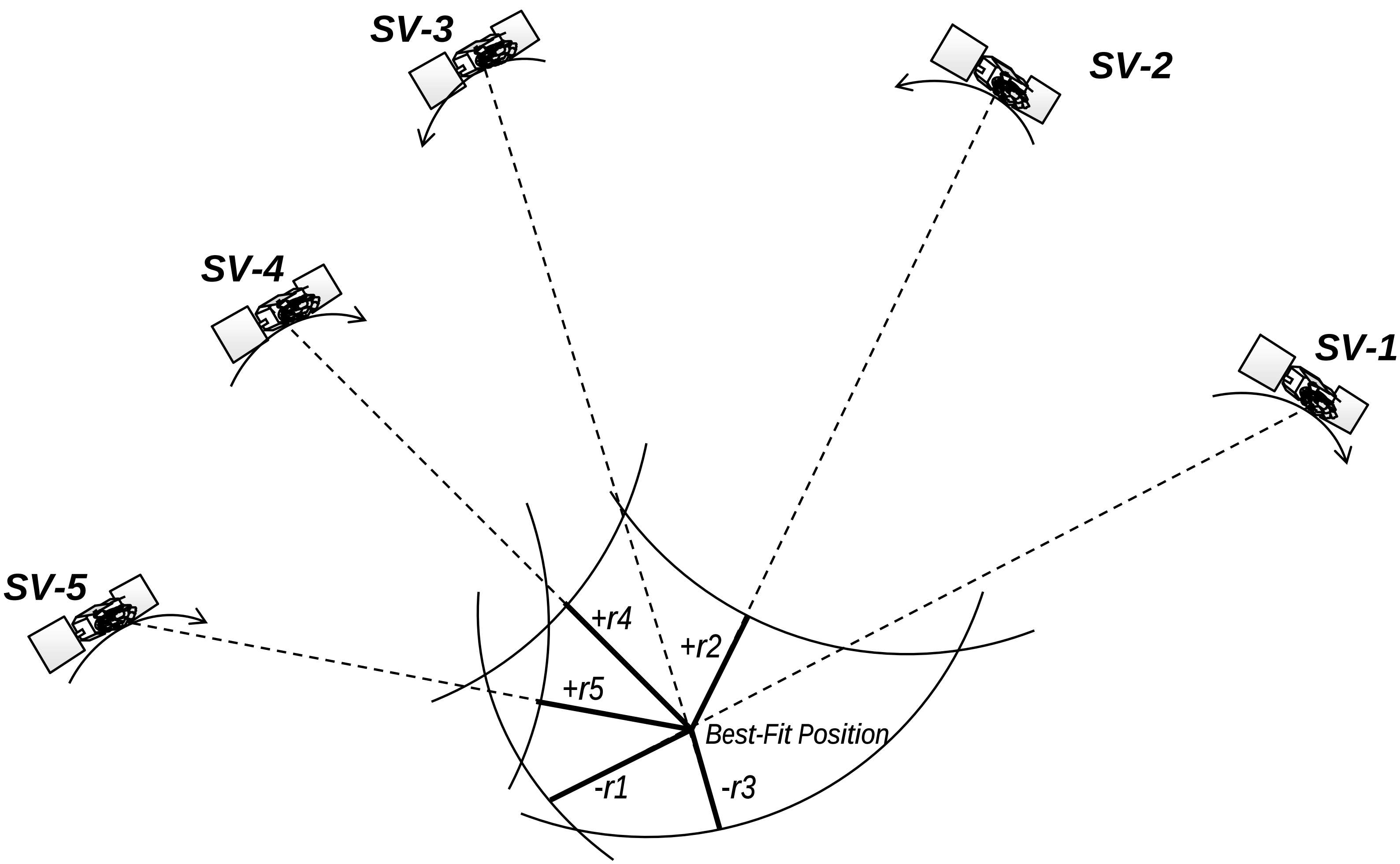
ndop = North DOP

edop = East DOP

vdop = Vertical DOP ***1-D Vertical DOP***

tdop = Time DOP

RMS Error from GNSS Position Range Residuals



$$rms = \pm \frac{(r_1^2 + r_2^2 + r_3^2 + \dots\dots\dots + r_n^2)^{1/2}}{n = \text{number of observations}}$$

GNSS Coordinate Quality (CQ) Values

$$
\begin{aligned}
CQ &= rms^2 . \text{ DOP Matrix} \\
&= [\, sdn^2, covn\_e, covn\_h, covn\_t \,] \\
&\qquad\quad [\, , sde^2 \;\; , cove\_h, cove\_t \,] \\
&\qquad\qquad\quad [\, , sdh^2 \;\; , covh\_t \,] \\
&\qquad\qquad\qquad\quad [\, , sdt^2 \;\;]
\end{aligned}
$$

where :

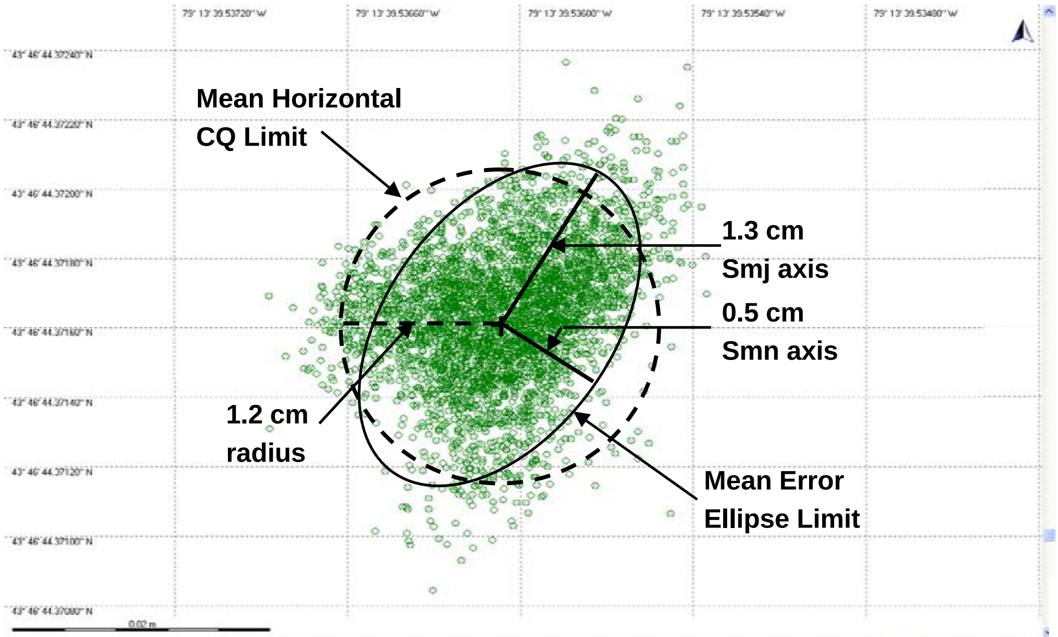
rms = Root Mean Square of Range Measurement Errors

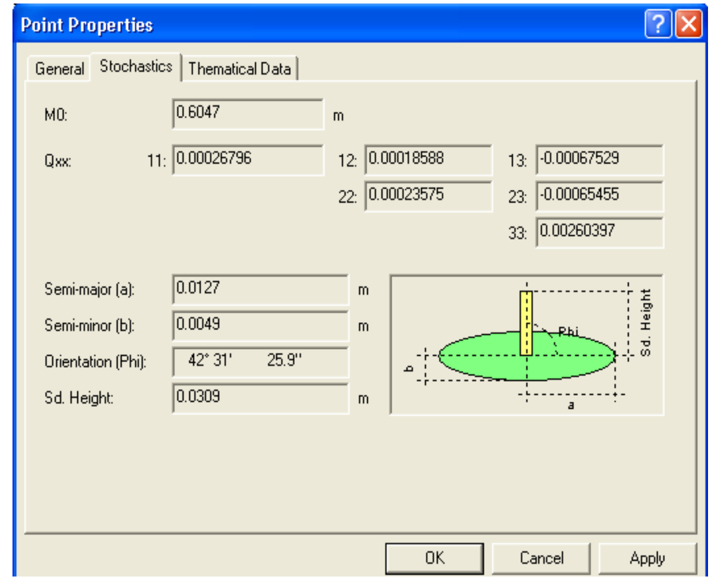
Horizontal CQ $= +/- (sdn^2 + sde^2)^{1/2}$ *(2d CQ)*

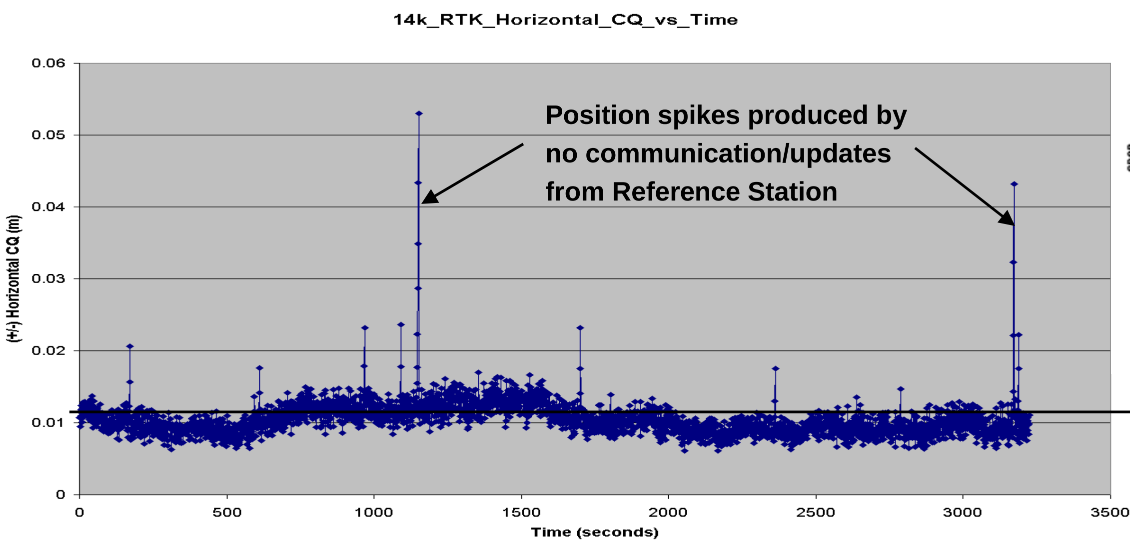
Vertical CQ $= +/- (sdh^2)^{1/2}$ *(1d CQ)*

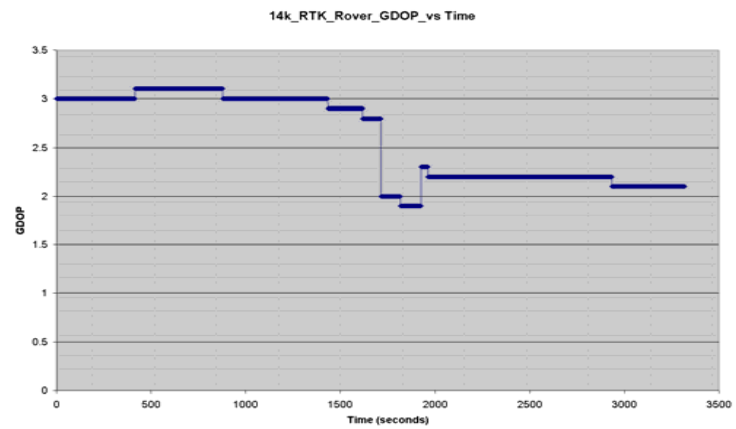
Spherical CQ $= +/- (sdn^2 + sde^2 + sdh^2)^{1/2}$ *(3d CQ)*

Short-Term RTK Horizontal Position Repeatability



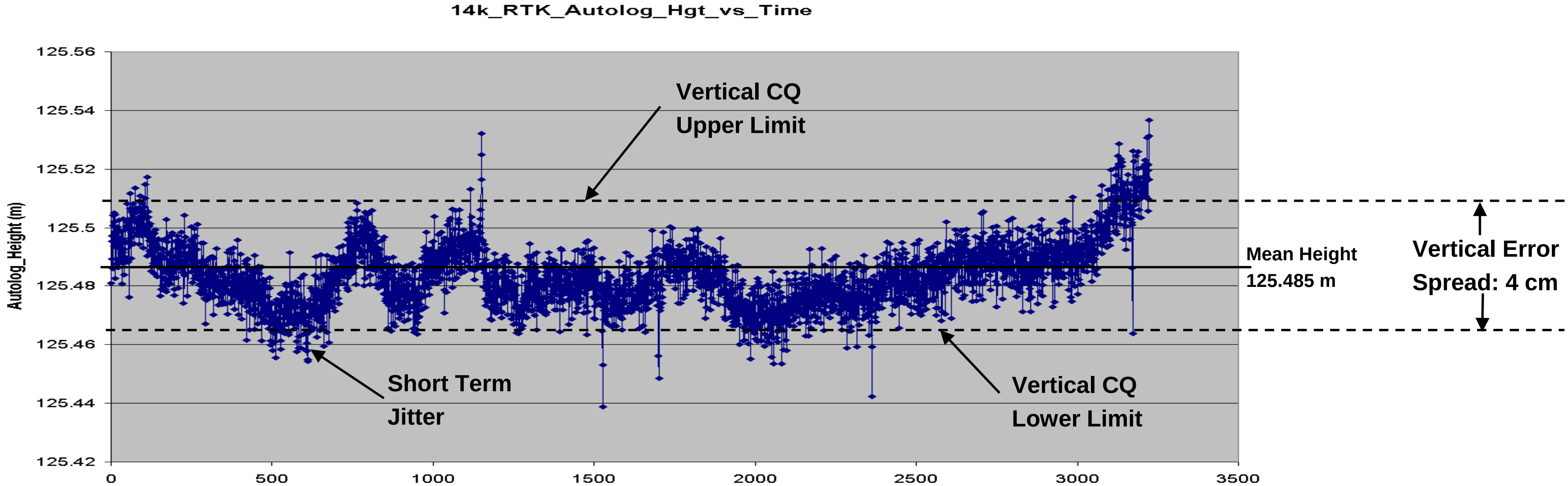


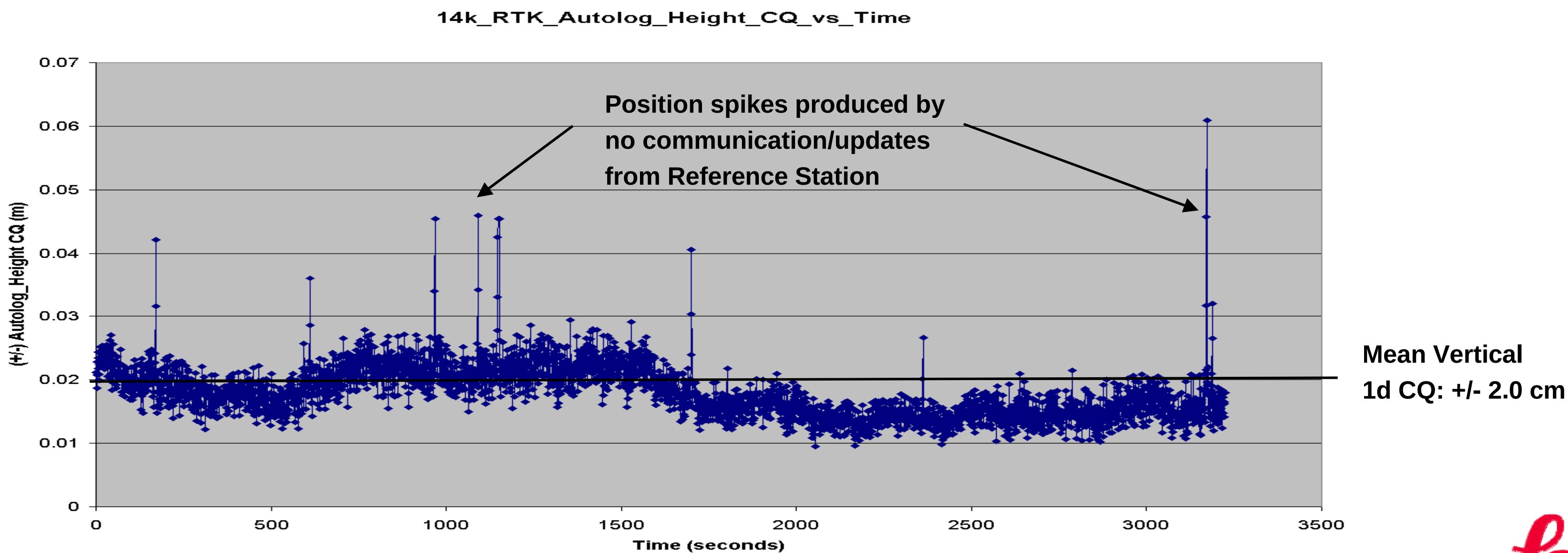




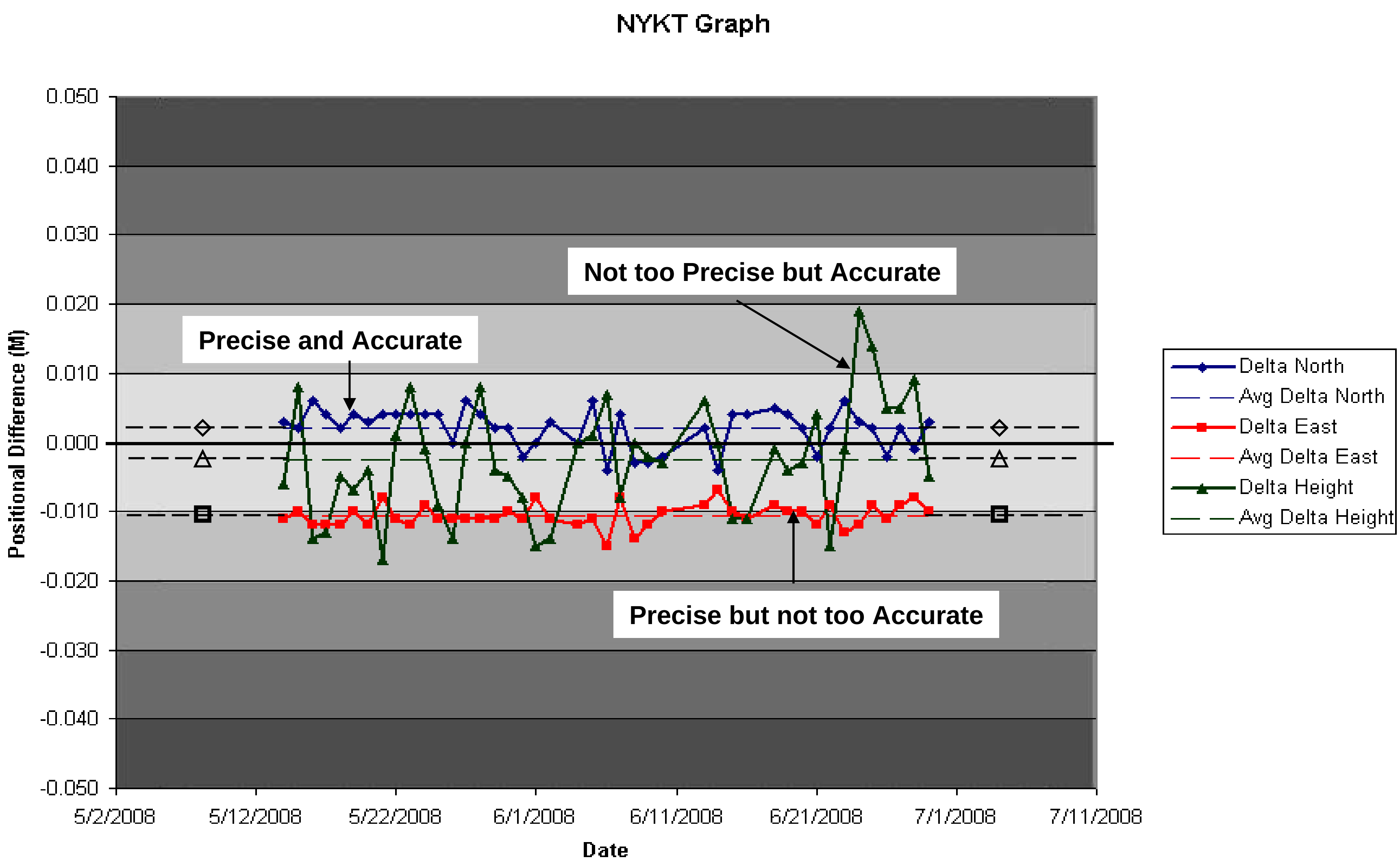
Mean Horizontal
2d CQ: +/- 1.2 cm

Short-Term RTK Vertical Position Repeatability

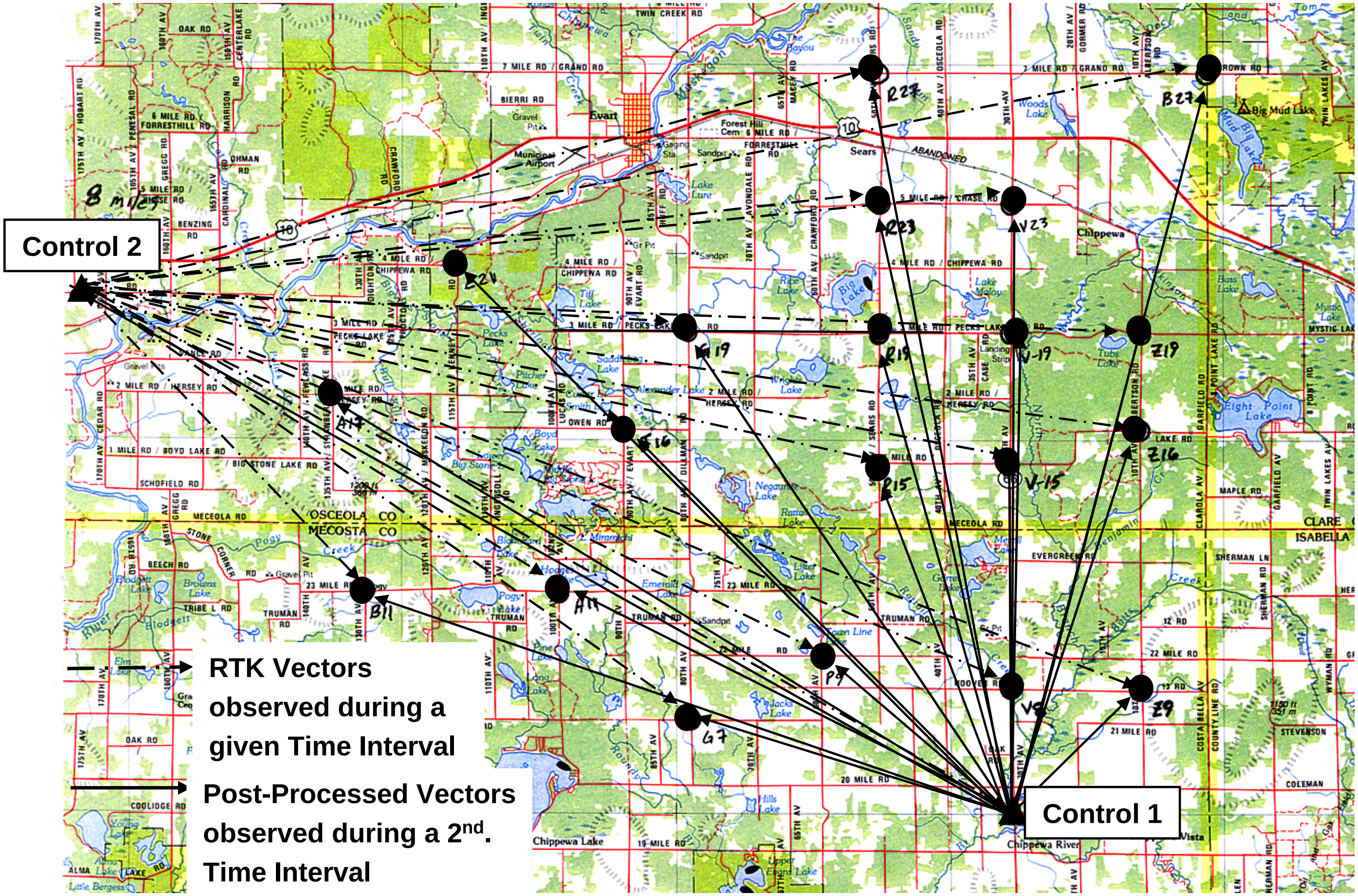




Long-Term Positioning Repeatability, Precision and Accuracy



Independent Site Re-Occupations at Different Times from Different Controls using Different Methods (RTK and Post-Processed)



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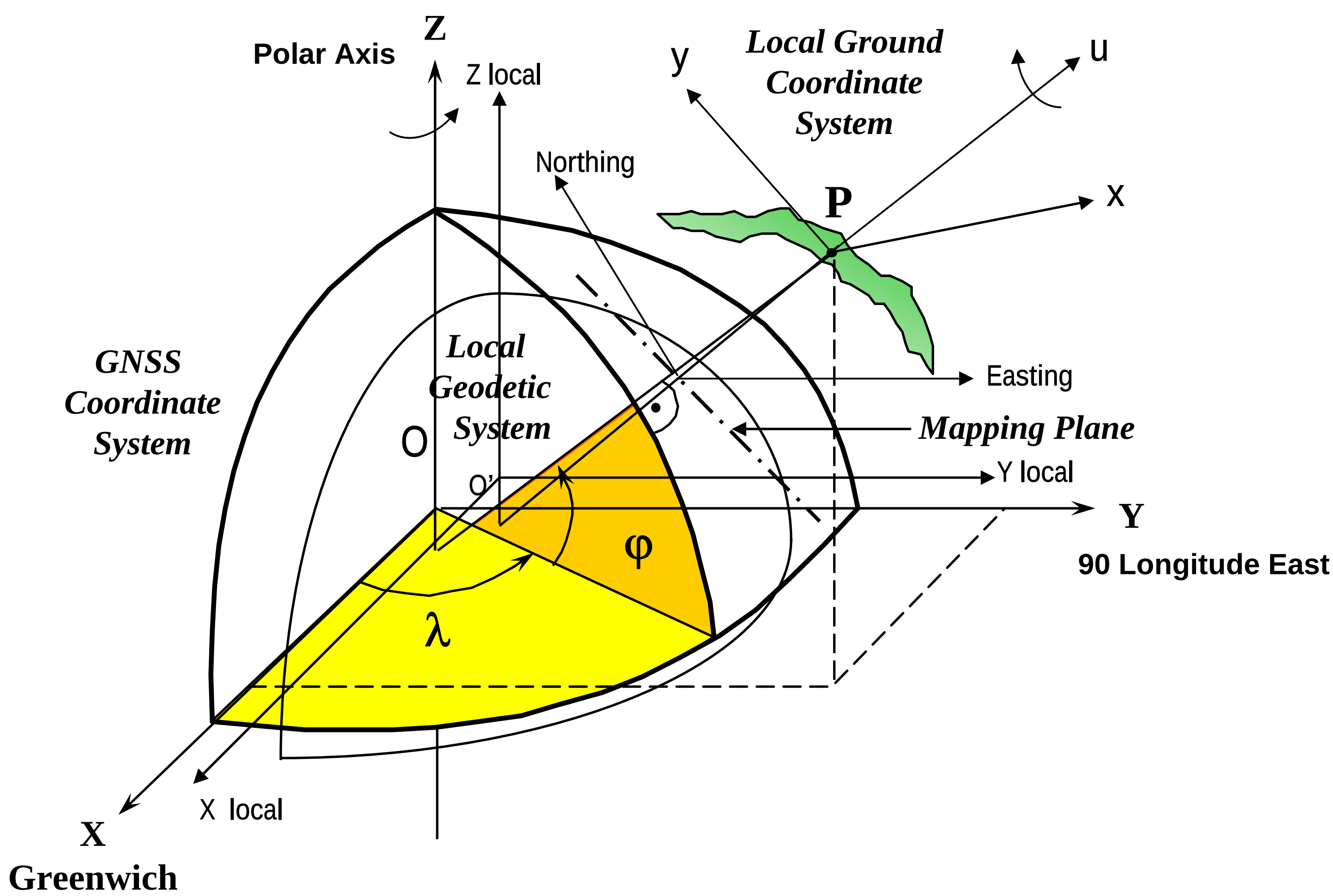
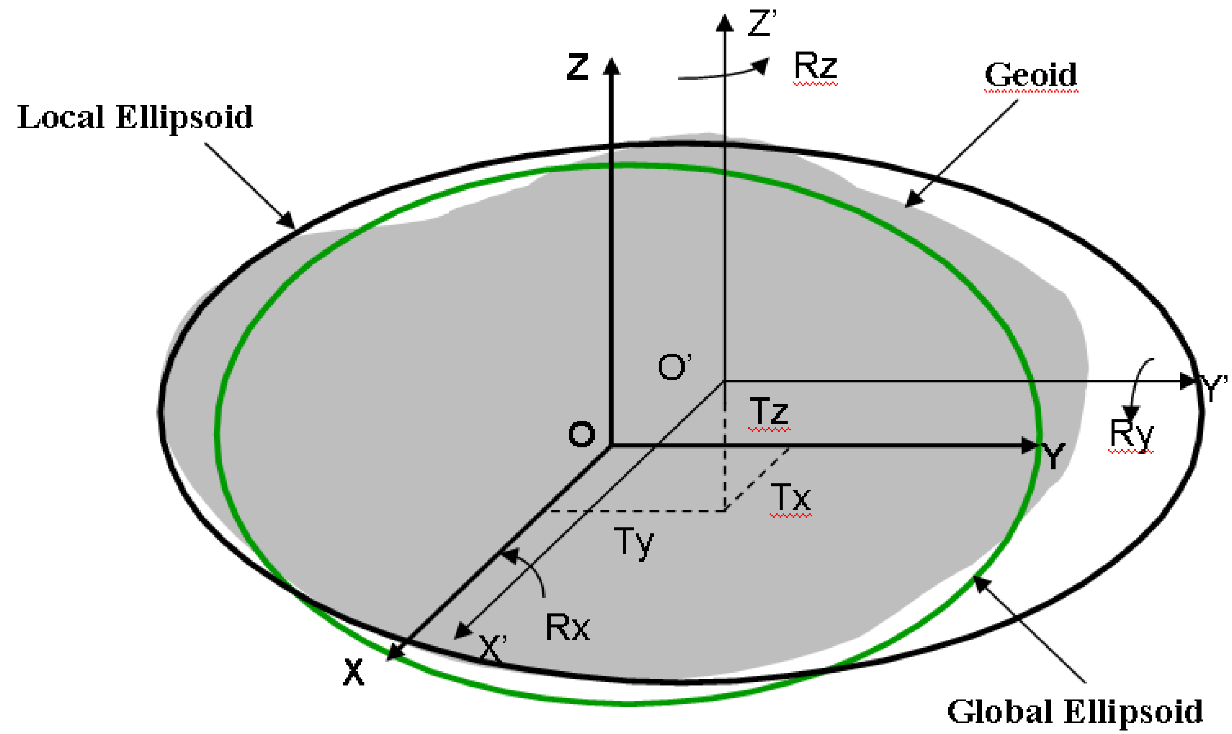
Transformation
of
GNSS Positions
to
Local Coordinate Systems

GNSS and Local Coordinate System Characteristics



3D Geodetic Datum Transformation (WGS84->NAD83 CSRS)



$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = Sc. \begin{bmatrix} Rx.\ Ry.\ Rz \\ \text{Rotation Matrix} \\ \text{(fully populated)} \end{bmatrix} . \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} Tx \\ Ty \\ Tz \end{bmatrix}$$

where: Tx, Ty & Tz : Translation components between the 2 origins
Rx, Ry & Rz : Rotation angles around the X, Y & Z axes
Sc : 3D Scale Factor between the 2 coordinate systems

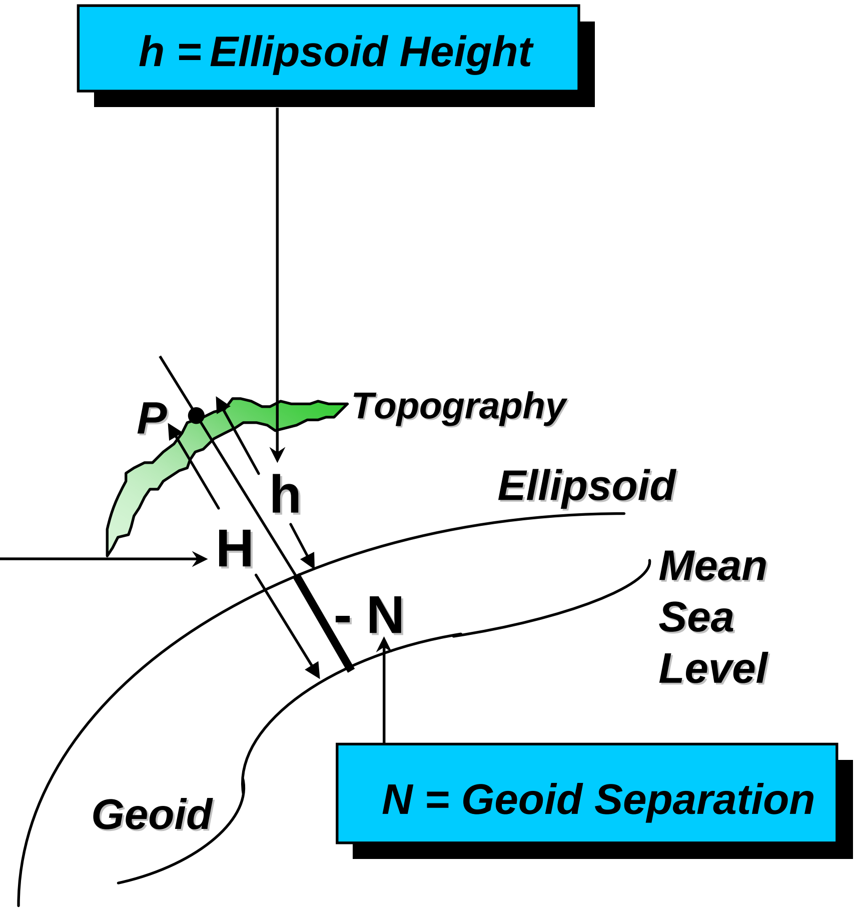
Geoid Model, Orthometric & Ellipsoid Heights

$$\boxed{h = H + (-N)}$$

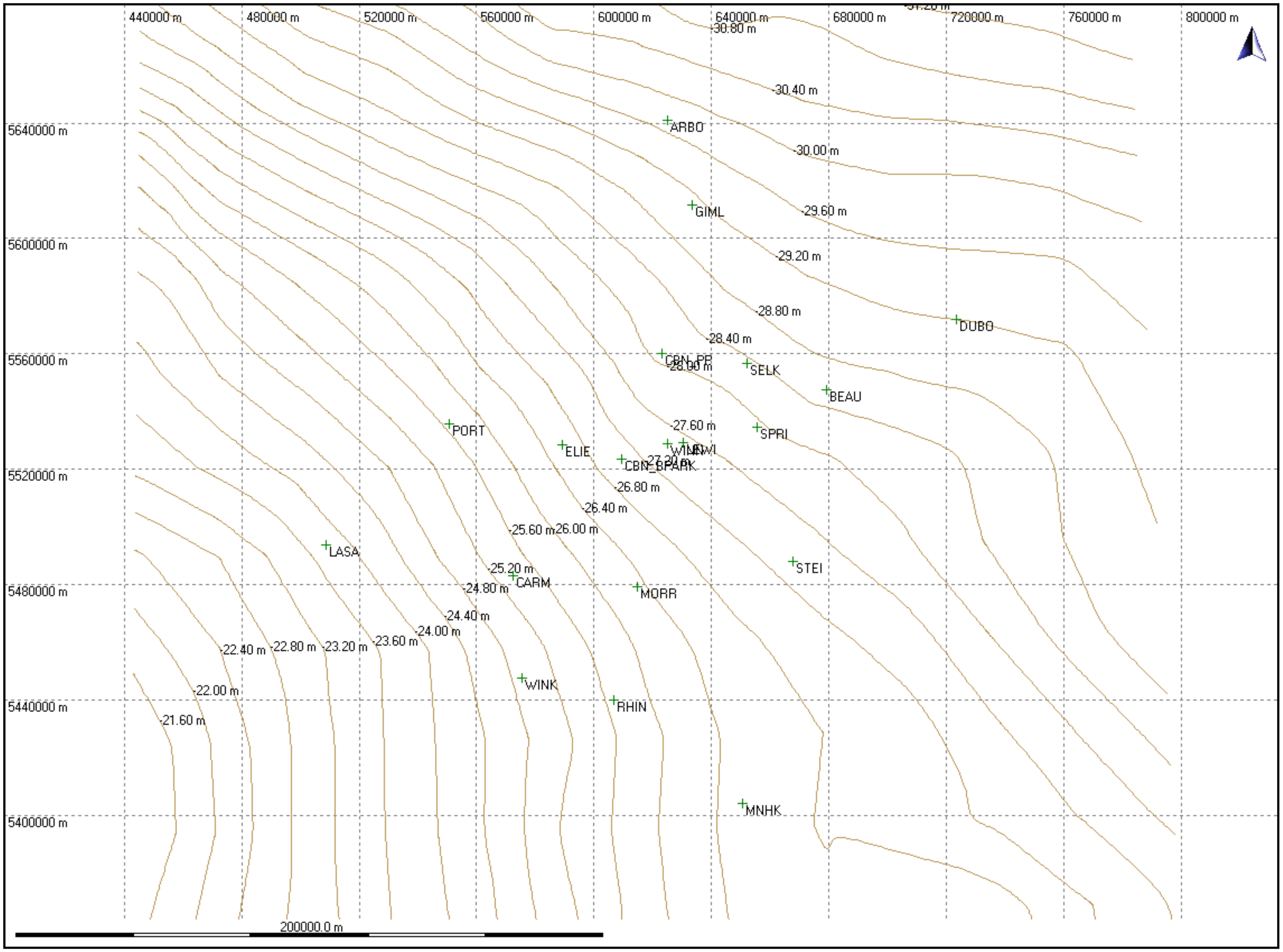
or

$$\boxed{H = h - (-N)}$$

H = Orthometric Height
(Mean Sea Level Height)



Geoid Separation Contours in Southern Manitoba



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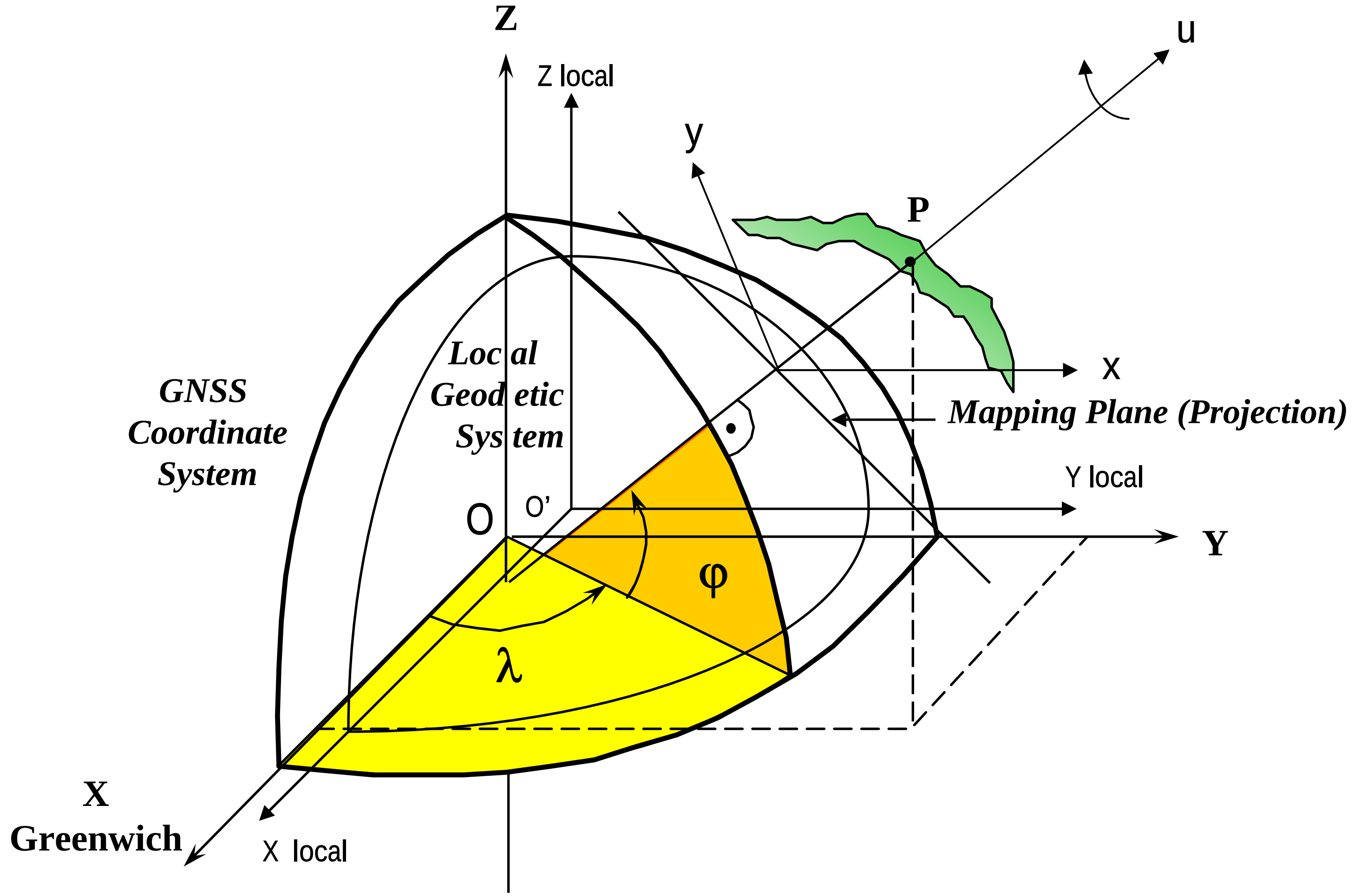
Transformation of GNSS to Local Mapping Grid Coordinates

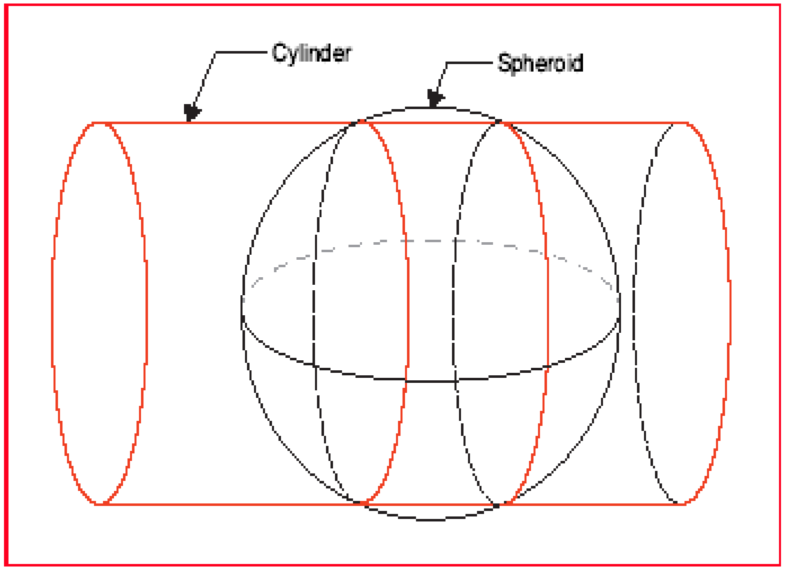


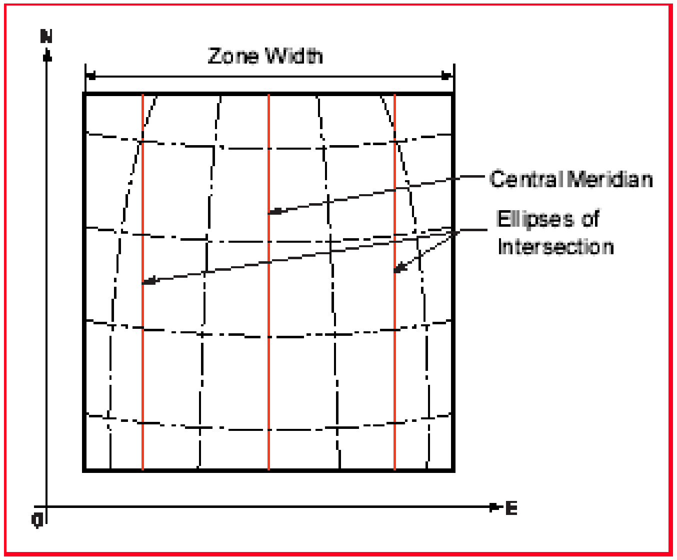
Characteristics of the Transverse Mercator Projection

A transverse cylinder intersects the surface of the spheroid along two small ellipses equidistant from the meridian through the center of the zone.

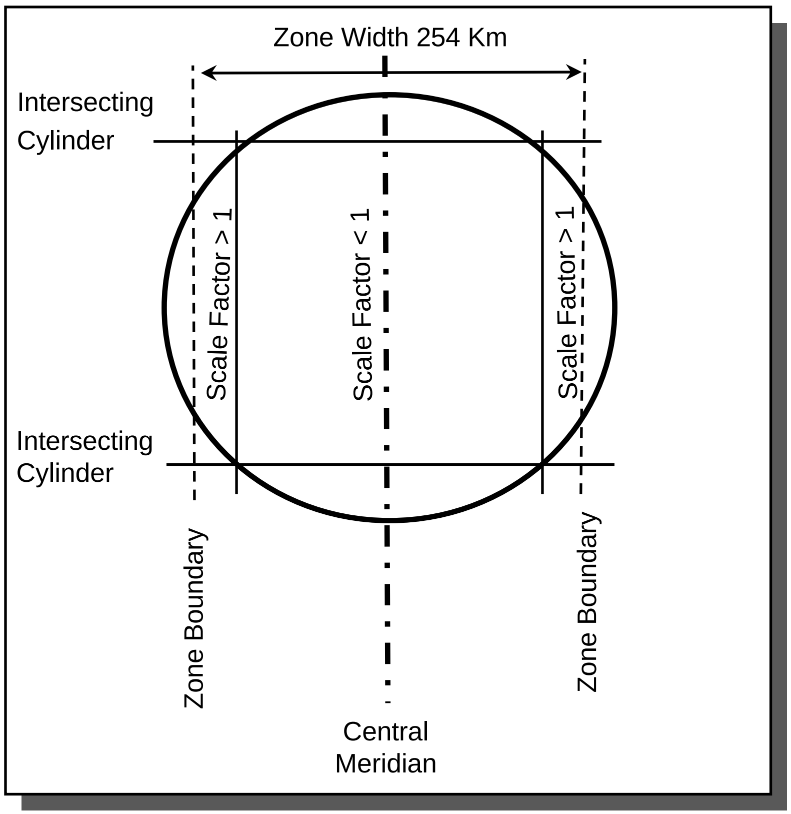
The transverse cylinder is defined by specifying this central meridian, plus the desired grid scale factor on the central meridian.

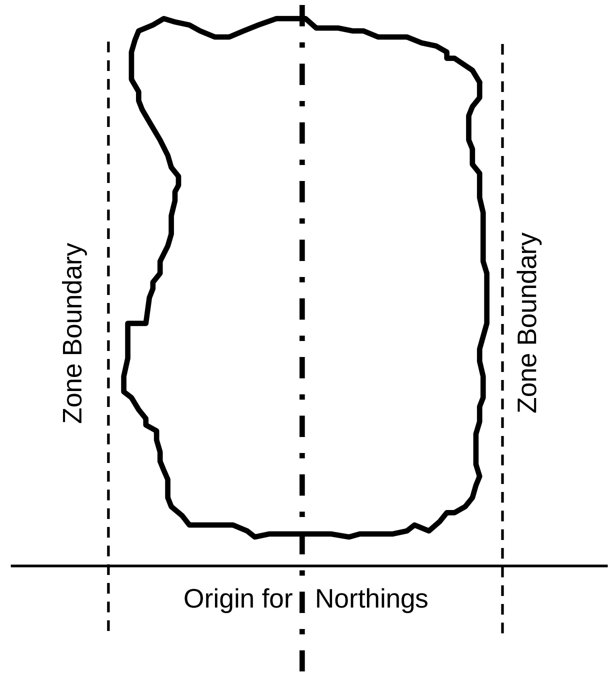
Scale remains the same along a North South (meridian) line.





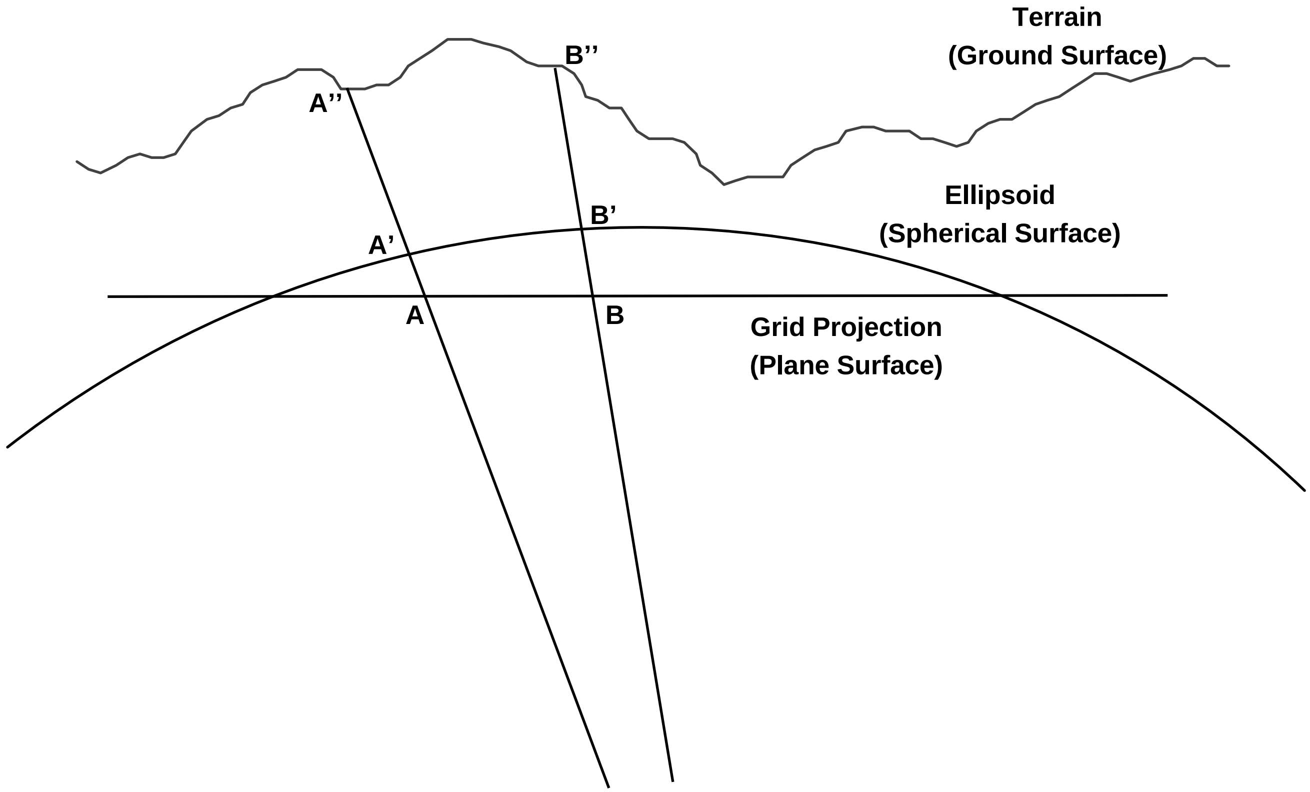
Characteristics of the Transverse Mercator Projection



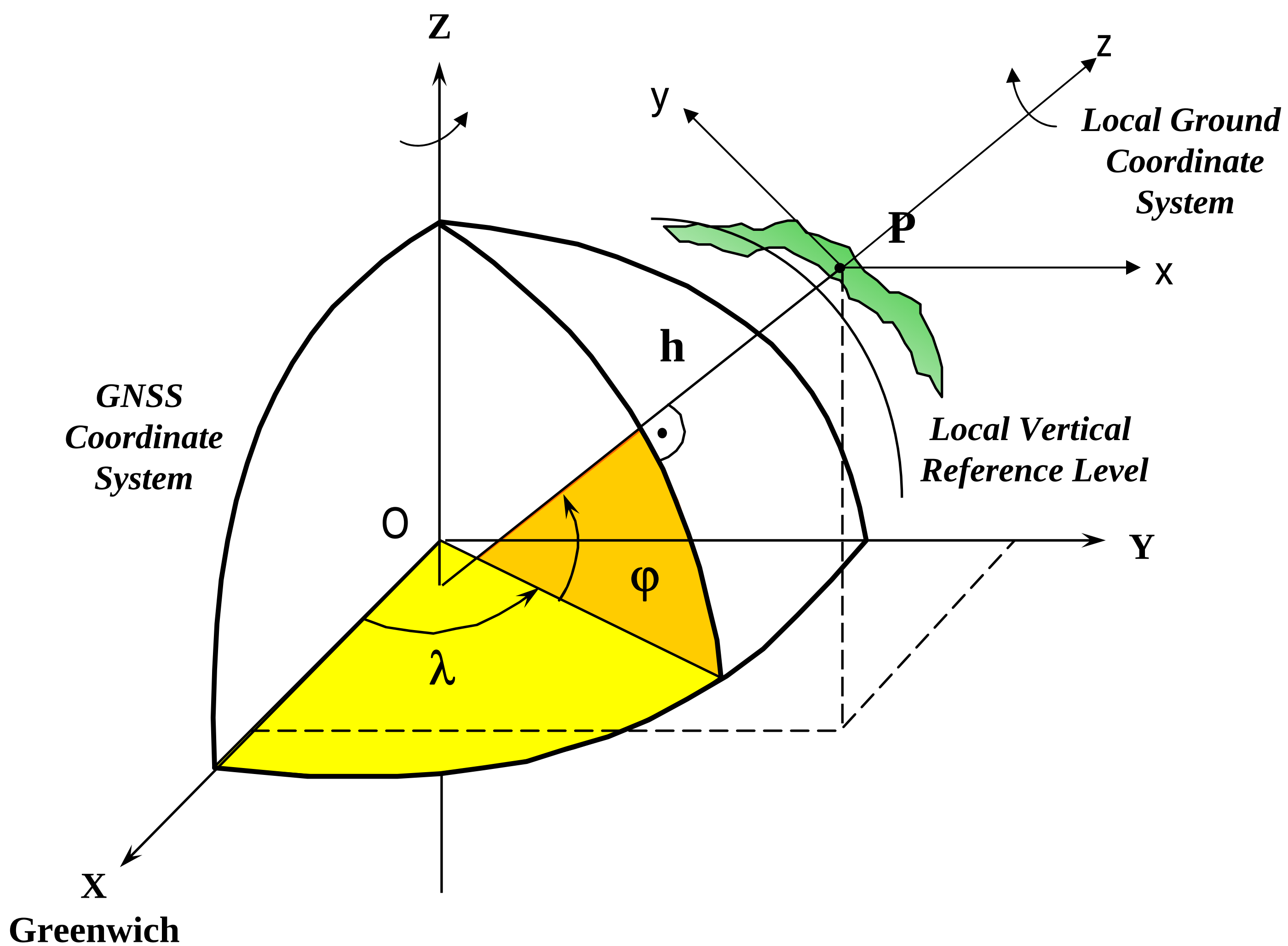


Scaling Mapping Grid Coordinates to Ground Level using Combined Scale Factor (CSF)

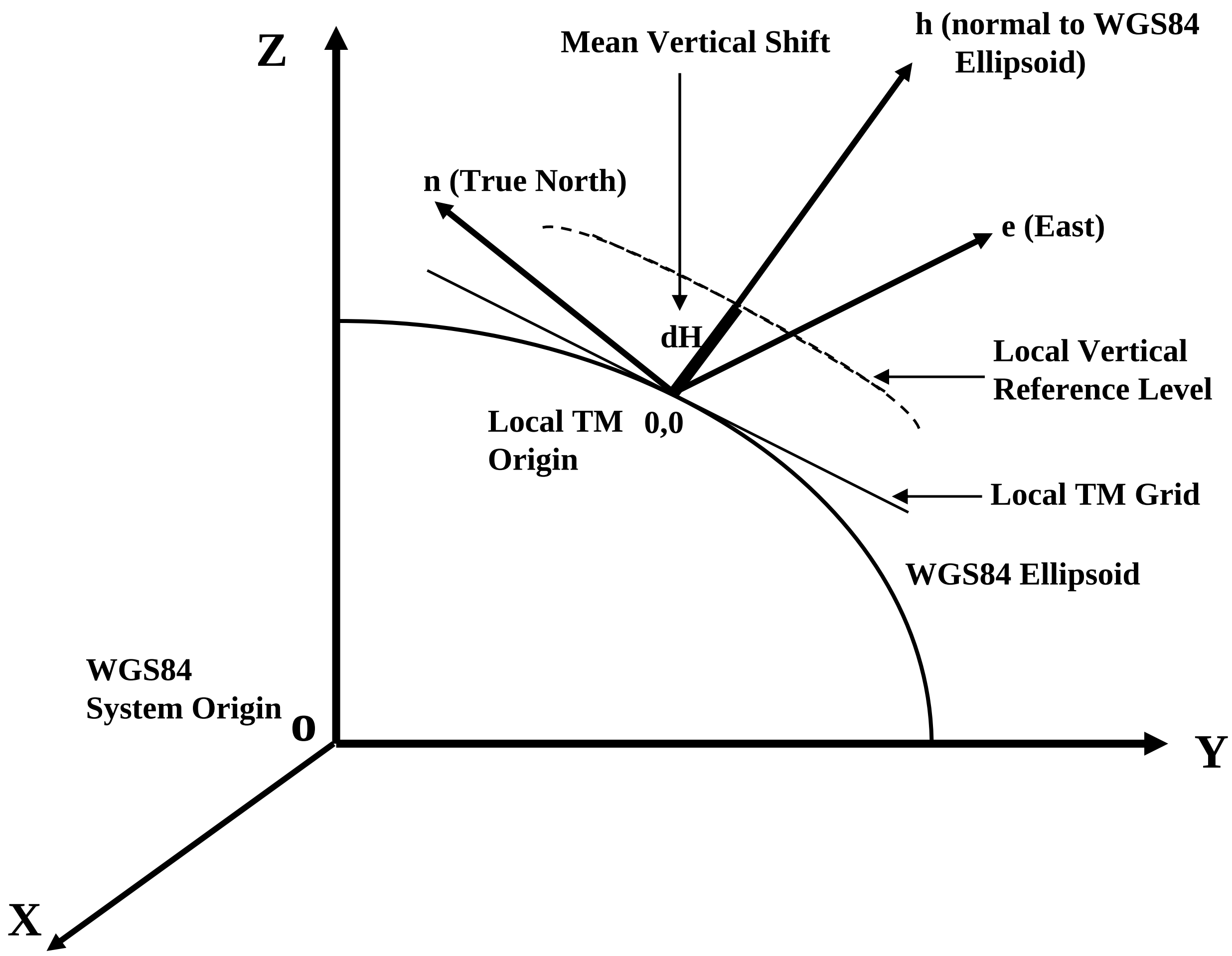
CSF = Projection Scale Factor x Ground Elevation Scale Factor



Transformation of GNSS to Local Ground Coordinates

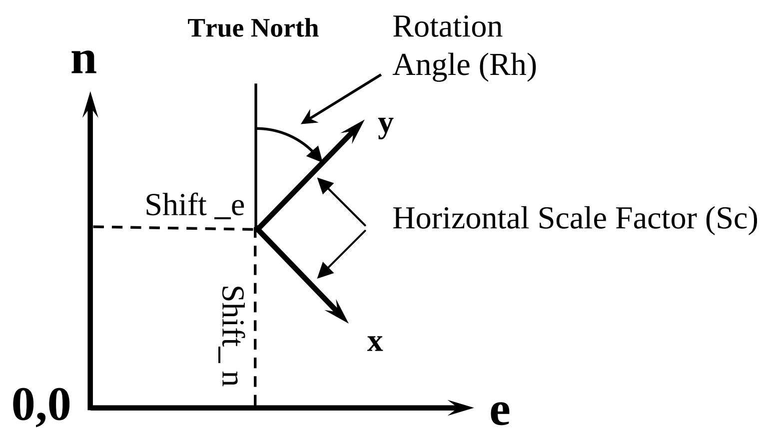


Transformation of GNSS to Local TM Grid

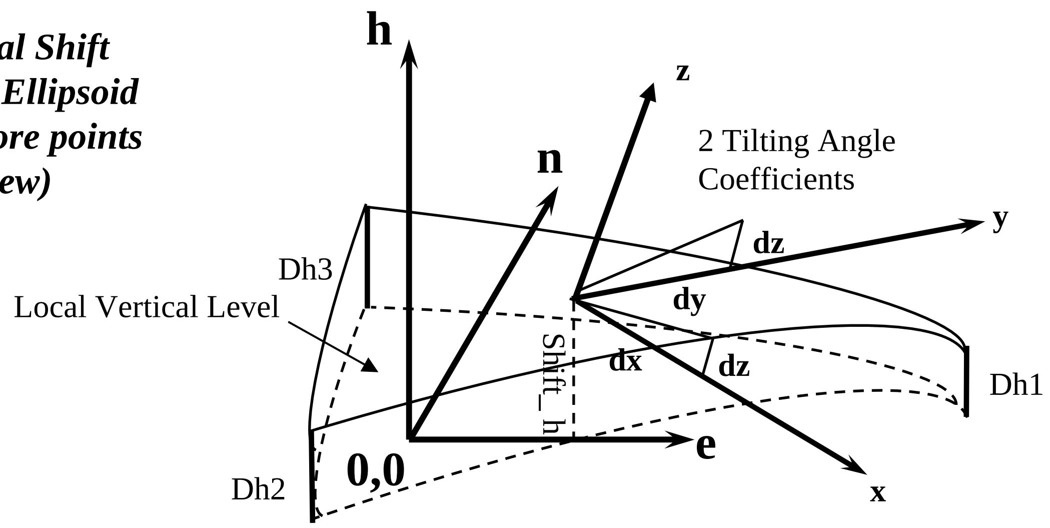


From Local TM Grid to Local (Arbitrary) Grid

2-D Helmert Horizontal Transformation (Plane View)



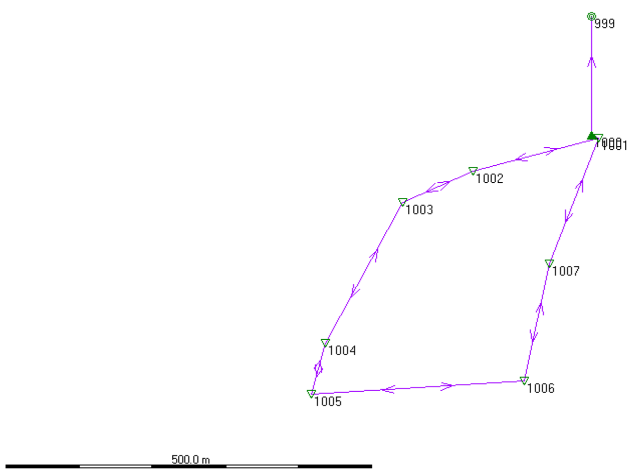
1-D Vertical Shift and Tilted Ellipsoid for 3 or more points (Profile View)

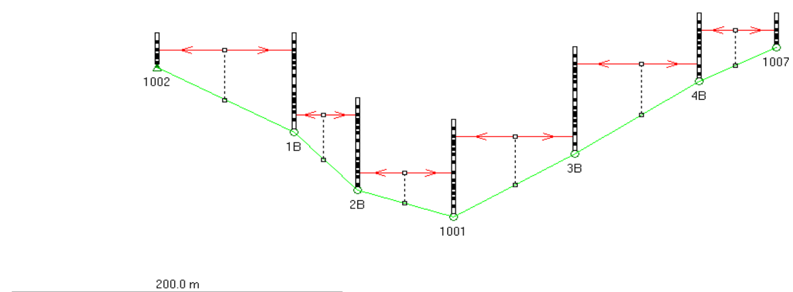


Intermixing GNSS Positions with Total Station and Level Data

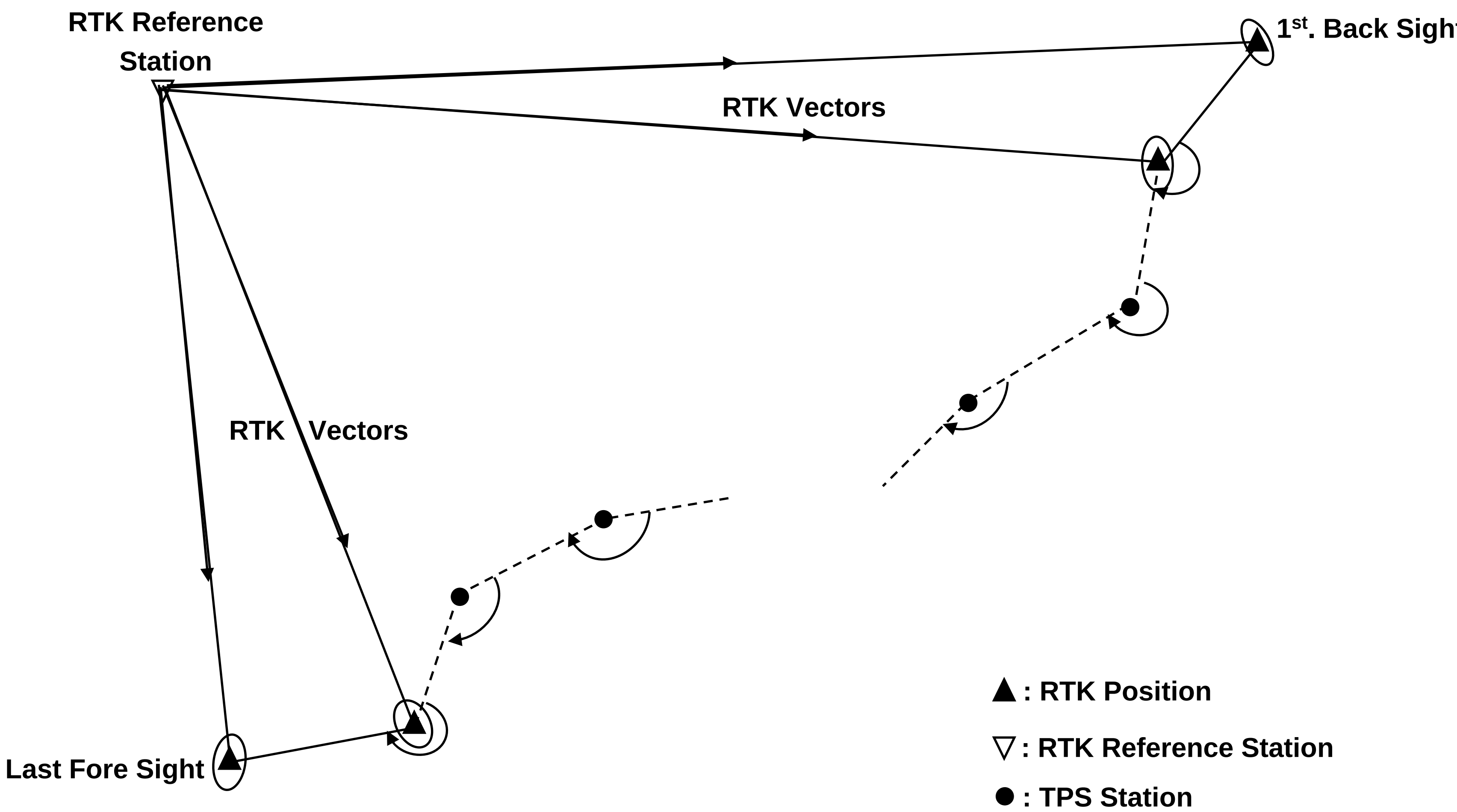
Setup Point Id	Backsight Point Id	Foresight Point Id	Hz	V	Slope Dist.	Horiz. Dist.	Height Diff.
1000	999	-	359° 59' 57.5"	90° 19' 09.5"	172.6404	172.6377	-0.8595
1000	-	1001	112° 38' 10.8"	89° 02' 01.2"	10.6650	10.6635	0.2378
1001	1000	-	292° 38' 16.3"	90° 57' 39.7"	10.6650	10.6635	-0.2368
1001	-	1002	254° 17' 04.4"	88° 50' 22.2"	178.3828	178.3461	3.6424
1002	1001	-	74° 16' 59.9"	91° 09' 46.7"	178.3826	178.3459	-3.6458
1002	-	1003	244° 50' 48.6"	89° 51' 43.5"	105.3752	105.3749	0.2453
1003	1002	-	64° 50' 38.9"	90° 08' 21.8"	105.3750	105.3747	-0.2464
1003	-	1004	207° 08' 35.8"	88° 51' 12.0"	230.3796	230.3333	4.3579
1004	1003	-	27° 08' 42.7"	91° 08' 57.6"	230.3806	230.3344	-4.3614
1004	-	1005	195° 07' 39.9"	88° 13' 43.7"	76.5550	76.5184	2.5860
1005	1004	-	15° 07' 40.9"	91° 46' 19.7"	76.5550	76.5184	-2.5865
1005	-	1006	86° 02' 13.7"	91° 37' 09.2"	291.5814	291.4653	-7.7397
1006	1005	-	266° 02' 15.3"	88° 23' 01.3"	291.5813	291.4650	7.7364
1006	-	1007	11° 25' 16.5"	89° 31' 05.3"	172.3317	172.3256	1.0307
1007	1006	-	191° 25' 16.8"	90° 19' 22.9"	172.3291	172.3263	-1.0397
1007	-	1001	20° 11' 06.2"	91° 15' 16.9"	194.8880	194.8414	-4.1429
1001	1007	-	200° 11' 12.8"	88° 44' 56.4"	194.8877	194.8411	4.1356
1001	-	1002	254° 17' 08.5"	88° 49' 02.5"	178.3834	178.3453	3.6382

Point Id	Back	Intm.	Intm. Type	Fore	Distance	Height	Point Class	Point Subclass	Code
1002						12.5352	Control	Fixed in Height	-
1002	0.9274				40.92				
1B				2.5077	41.69				
1B				2.4667	41.69				
1B						28.8996	Measured	None	-
1002	0.8862				40.94				
1B	0.7396				18.34				
2B				2.1415	20.75				
2B				2.0916	20.77				
2B						27.4978	Measured	None	-
1B	0.6898				18.31				
2B	1.0118				28.31				
1001				1.6686	29.56				
1001				1.6422	29.55				
1001						26.8410	Measured	None	-
2B	0.9854				28.31				
1001	2.3125				37.58				
3B				0.7628	36.09				
3B				0.7269	36.10				
3B						28.3908	Measured	None	-
1001	2.2768				37.57				
3B	2.3649				40.07				
4B				0.6040	34.84				
4B				0.6322	34.85				

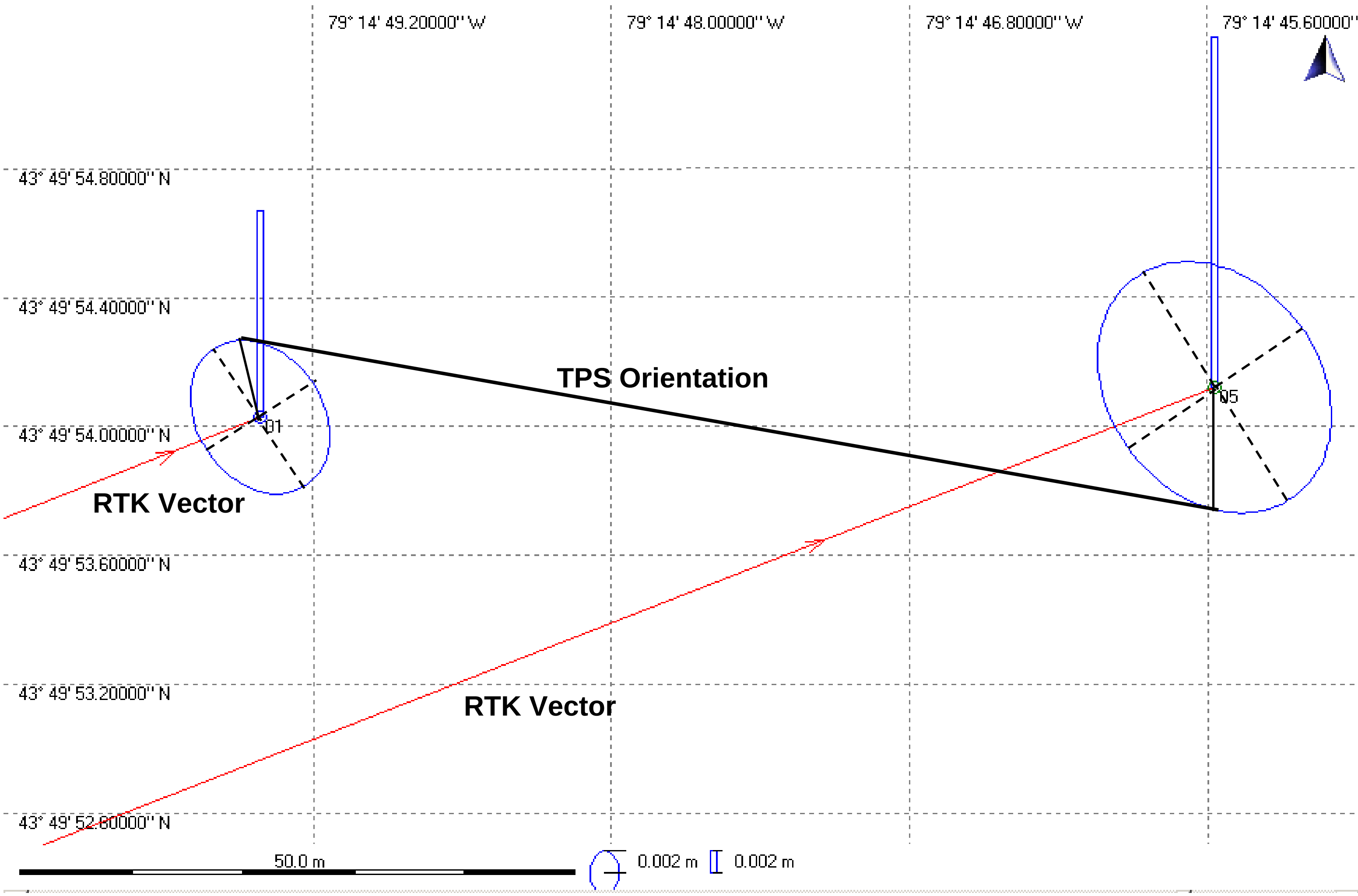




TPS Traversing from GNSS RTK Positions

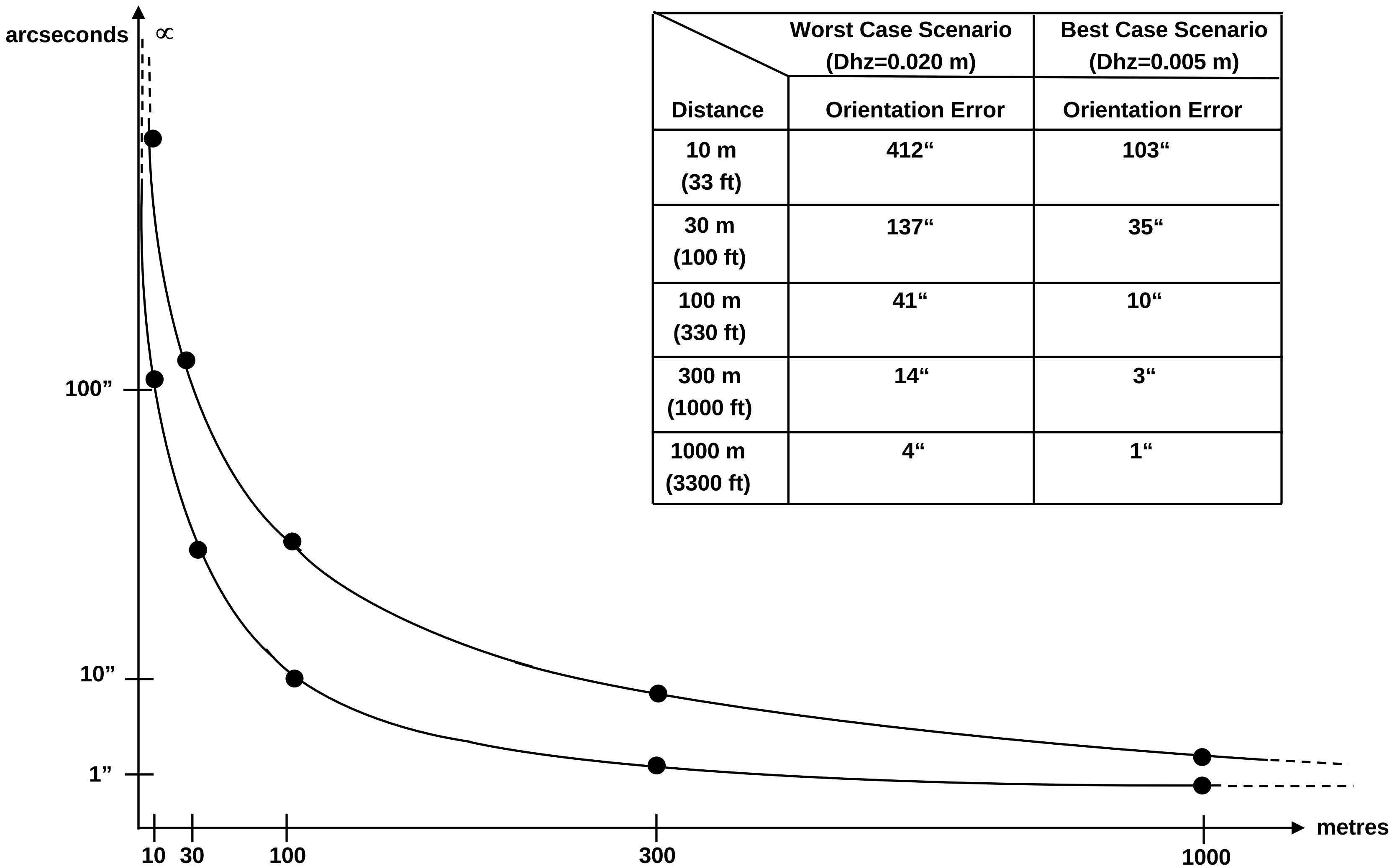


TPS Orientation from RTK Positions



- when it has to be **right**

TPS Orientation Errors from RTK Positions Errors at Different Distance Separations



Distance	Worst Case Scenario (Dhz=0.020 m) Orientation Error	Best Case Scenario (Dhz=0.005 m) Orientation Error
10 m (33 ft)	412“	103“
30 m (100 ft)	137“	35“
100 m (330 ft)	41“	10“
300 m (1000 ft)	14“	3“
1000 m (3300 ft)	4“	1“

Summary and Conclusion

Summary

- ***A brief review of GNSS Characteristics with its Current Status have been presented. Over 50 satellites are operational and provide 10 or more visible Satellites most of the time in most areas of the world.***
- ***GNSS Measurement and Positioning Processes have been examined for their application in Land Surveying and Engineering with a special emphasis on Phase Measurements in Relative Positioning Baseline Vector determination.***
- ***Real-Time GNSS Positioning Methods have been described for Single Base and Network RTK Baseline Vector solutions using Wireless and Internet Communication.***
- ***GNSS Coordinate Quality Indicators based on Satellite Geometry and Measurement RMS errors for Short Term and Long Tern Positioning Repeatability with Independent Site Re-Occupations have been examined as ways to assess GNSS Positioning Reliability.***
- ***Different Transformations of GNSS Positions to Local Coordinate Systems have been presented for converting GNSS coordinates to Local Geodetic Datum such as NAD83 CSRS, to UTM Mapping Grid Projection and Local Ground Coordinate Systems so that GNSS positions can be properly inter-mix with Terrestrial Data from Total Station and Level Instruments.***

Conclusion

- *GNSS Positioning is very versatile and sophisticated.*
- *A lot of background knowledge and various skills are required to fully exploit GNSS Positioning capabilities.*
- *We should be all very grateful to the people that have developed the fundamentals of Satellite Positioning and Navigation during the past few centuries up to the more recent organizations that have designed, developed and are maintaining these systems in operation with the participation of emerging computer and communication technologies.*
- *It is up to us to use it responsibly.*