Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:

1. 20 marks
2. 20 marks
3. 20 marks
4. (a) 6 marks, (b) 14 marks
5. (a) 10 marks, (b) 10 marks
6. 20 marks
7. 20 marks
8. 20 marks
1. Find the general solution of the differential equation \( x^2y' - 2xy' + 2y = (1 - 2x)x^2e^{-2x} \).
   Note that ' denotes differentiation with respect to \( x \).

2. Find the general solution, \( z(t) \), of the differential equation \( z'' + 4x = 3\cos 2t + 4\cos 3t \).
   Note that ' denotes differentiation with respect to \( t \).

3. Find the maximum and minimum values of \( f(x, y, z) = 4x + y^2 + 2z^2 \) over the ellipsoid \( x^2 + 3y^2 + z^2 = 2 \).

4. Let \( x = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \) and \( A = \begin{pmatrix} 1 & 1 & 6 \\ -1 & 2 & -2 \\ 1 & -1 & 0 \\ 1 & 1 & 2 \end{pmatrix} \)
   (a) Show that \( x \) is an eigenvector of \( A \) and find the associated eigenvalue.
   (b) Show that \( 3 \) is an eigenvalue of \( A \) and find an associated eigenvector.

5. Let \( f(x, y, z) = x^2 + y^2 + z^2 + 2y - 3x \), and let \( g(x, y, z) = 3x + y^2 - z^2 \).
   (a) Find an equation for the tangent plane to the surface \( g(x, y, z) = 9 \) at the point \( (3, -1, 1) \).
   (b) Find the line tangent to the intersection of the surfaces \( f(x, y, z) = 0 \) and \( g(x, y, z) = 9 \) at the point \( (3, -1, 1) \).

6. Evaluate the surface integral \( \iiint_S F \cdot dS \) where \( F(x, y, z) = xzi - 2yj + 3zk \) and \( S \) is the surface of the region bounded above by the paraboloid \( z = 4 - x^2 - y^2 \) and below by the plane \( z = 0 \).

7. Find the work done by the field \( F(x, y, z) = x^2i + yj - zk \) in moving a particle from the point \( (0, 2, 0) \) to the point \( (3\pi, 0, 2) \) along the path \( x = 6t, y = 2\cos t, z = 2\sin t \).

8. Evaluate the surface integral \( \iiint_S x^2yz \, dA \) where \( S \) is the portion of the cylinder \( x^2 + y^2 = 4 \) with \( 0 \leq z \leq 4 \) and \( y \geq 0 \).