NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. The examination is an OPEN BOOK EXAM.

3. Candidates may use any non-communicating calculator.

4. All problems are worth 25 marks. One problem from each of sections A, B, and C must be attempted. A fourth problem from any section must also be attempted.

5. Only the first four questions as they appear in the answer book will be marked.

6. State all assumptions clearly.
Section A: Fluid Mechanics


(a) [15 marks] Starting with the following form of the Navier-Stokes equation

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_\theta \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \frac{\partial^2 u_z}{\partial r^2} + \nu \frac{\partial^2 u_z}{\partial \theta^2} + \nu \frac{\partial^2 u_z}{\partial z^2}$$

show through clear assumptions and logical analysis that the relationship of the axial velocity at any point ($u_z$) to the maximum velocity ($u_{z,max}$) is:

$$\frac{u_z}{u_{z,max}} = 1 - \frac{r^2}{R^2}$$

(b) [5 marks] Also show that the volumetric flow rate is given by:

$$\dot{V} = \pi \left(\frac{-\Delta P}{8\mu L}\right) R^4$$

A2 [25 marks overall] A decrease of 0.71 m in head of liquid is required to drive a liquid (density $= 984$ kg/m$^3$) through a smooth horizontal pipeline (inside diameter $= 0.55$ m, length $= 658$ m) at a volumetric flow rate of 0.206 m$^3$/s. Calculate the following:

(a) [5 marks] The pressure drop per unit length of pipe in Pa/m.

(b) [5 marks] The Fanning friction factor.

(c) [5 marks] The total force exerted by friction on the pipeline.

(d) [5 marks] The kinematic viscosity of the liquid in cSt.
Section B: Heat Transfer

B1 [25 marks overall] Consider a hollow cylindrical heat transfer medium of length $L$ having inside and outside radii of $r_i$ and $r_0$ with corresponding temperatures $T_i$ and $T_0$. If the thermal conductivity varies linearly with temperature according to:

$$k(T) = k_0(1 + \beta T)$$

starting with the appropriate form of Fourier's law of heat conduction show that the steady-state rate of heat transfer in the radial direction $\dot{Q}_r$ is given by:

$$\dot{Q}_r = \frac{2\pi Lk_0}{\ln(r_0/r)} \left[1 + \frac{\beta}{2} (T_i + T_0)\right] (T_i - T_0)$$

B2 [25 marks overall] A large slab of pure aluminium is of half-thickness $L = 0.5$ m. It is initially at a uniform temperature of 200°C and is suddenly exposed to a convection environment in which the fluid temperature is 50°C and the heat transfer coefficient is 948 W/m²K. Calculate the following:

(a) [15 marks] The temperature of the slab 200 mm from one of the surfaces after 30 minutes; and

(b) [10 marks] How much heat is lost from the slab during this time?

Useful charts are shown in Figs B1 and B2.

Useful data for aluminium:

$\rho = 2,702$ kg/m³; $c_p = 903$ J/kg K; $k = 237$ W/m K; and $\alpha = 97.1 \times 10^{-6}$ m²/s
Fig. B1: The transient temperature distribution in a slab at six positions; $x/L = 0$ is the centre, and $x/L = 1$ is the outside boundary.
Fig. B2: The heat removal from suddenly cooled bodies as a function of \( \bar{h} \) and time
Section C: Mass Transfer

C1 [25 marks overall] A permeable thick-walled hollow sphere has an internal radius of $R_1$ and an external radius of $R_2$. The inner surface is maintained at a constant concentration $C_{Ai}$ and the outer surface is maintained at a constant concentration $C_{Ao}$. Starting with the following form of the continuity equation

$$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial C_A}{\partial \phi} =$$

$$\dot{\Omega}_{A,\theta} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 D \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( D \sin \theta \frac{\partial C_A}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( D \frac{\partial C_A}{\partial \phi} \right)$$

show through clear assumptions and logical analysis that the steady-state concentration profile for component $A$ within the wall is:

$$C_A = C_{Ai} + \{(C_{Ao} - C_{Ai})R^2\left[(r - R_1)/R\right]/(R_2 - R_1)\}$$

C2 [25 marks overall] A piece of porous glass tubing is used as a diffusion cell to measure the diffusion coefficient of an air-gas mixture. The inside diameter and the outside diameter of the cell are 1 mm and 4 mm. It was found that with a difference in mole fraction of 10% at 25°C and 1 atmosphere pressure, the molar flow rate was $4 \times 10^{-6}$ mol/s per cm of length. Find the diffusion coefficient in m$^2$/s.