National Exams May 2010

07-Elec-A2, Systems & Control

3 hours duration

NOTES:

1. This is a CLOSED BOOK EXAM. Candidates are allowed to use a handwritten, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet has to be signed and submitted together with the examination paper.

2. Candidates may use one of two types of calculators, the Casio or Sharp approved models.

3. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

4. Five (5) questions constitute a complete paper. YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2. Choose three (3) more questions out of the remaining six. Each question is of equal value. Weighting of topics within a question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.

5. PLEASE WRITE ANSWERS DIRECTLY IN THIS EXAM PAPER – DO NOT USE EXAM BOOKS. If necessary, you may write on the backside of the pages as long as you write the final answers in the space indicated, and point to where the full calculations are.

<table>
<thead>
<tr>
<th>YOUR MARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>QUESTIONS 1 AND 2 ARE COMPULSORY:</td>
</tr>
<tr>
<td>Question 1</td>
</tr>
<tr>
<td>Question 2</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:</td>
</tr>
<tr>
<td>Question 3</td>
</tr>
<tr>
<td>Question 4</td>
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<td>Question 5</td>
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<td>Question 6</td>
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<tr>
<td>Question 7</td>
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<tr>
<td>Question 8</td>
</tr>
<tr>
<td>TOTAL:</td>
</tr>
</tbody>
</table>
Question 1 (Compulsory)

Block Diagrams, Transfer Functions, Mason's Gain Formula, Second Order Model, Dominant Poles, Stability and Gain Margin, Proportional Control

PART A (5 marks)

Consider two systems represented by two SIMULINK diagrams shown below. Identify the important difference between the two of them, and show how it will affect the Mason's Gain formula used to find transfer functions of the two systems. Find both transfer functions, \( G_1(s) \) and \( G_2(s) \).

\[
G_1(s) = \frac{Y(s)}{R(s)} = \frac{1}{s} + \frac{10}{s+1}
\]

\[
G_2(s) = \frac{Y(s)}{R(s)} = \frac{1}{s} + \frac{10}{s+1}
\]
PART B (5 marks)

- Consider a certain closed loop control system, which is TYPE ONE. It is a second order system with two complex conjugate poles located at: $-3 \pm j3$. Write its transfer function below:

\[ G_1(s) = \]

- Consider a certain closed loop control system, which is TYPE ONE. It is a second order system with two complex conjugate poles located at: $-3 \pm j3$ and a zero at $-15$. Write its transfer function below:

\[ G_2(s) = \]

- Consider a certain closed loop control system, which is TYPE ONE. It is a second order system with two complex conjugate poles located at: $-3 \pm j3$ and a zero at $+15$. Write its transfer function below:

\[ G_3(s) = \]

- Consider a certain closed loop control system, which is TYPE ONE. It is a third order system with two complex conjugate poles located at: $-3 \pm j3$ and a pole at $-15$. Write its transfer function below:

\[ G_4(s) = \]
PART C (5 marks)

1) Which of the following is a transfer function for the system below?

- $G(s) = \frac{4}{s^2 + 6s + 12}$
- $G(s) = \frac{4(s + 3)}{s^2 + 6s + 8}$
- $G(s) = \frac{4}{s^2 + 6s + 8}$
- $G(s) = \frac{2(s + 4)}{s^2 + 6s + 12}$

2) Which of the following is a transfer function for the system below?

- $G(s) = \frac{4}{s^2 + 6s + 12}$
- $G(s) = \frac{4(s + 3)}{s^2 + 6s + 8}$
- $G(s) = \frac{4}{s^2 + 6s + 8}$
- $G(s) = \frac{2(s + 4)}{s^2 + 6s + 12}$

3) Consider the plot below showing a system response at $K = 12$. What is that system Gain Margin, $G_m$, if the operational gain is $K_{op} = 3$?

![Closed Loop Response at K = 12](image)

$G_m =$

4) Consider a system with the transfer function as shown below. What is its DC gain?

$G(s) = \frac{45(s + 2)}{(s + 4)(s^2 + 3s + 25)}$

- $K_{dc} = 45$
- $K_{dc} = 0.9$
- $K_{dc} = 90$
- $K_{dc} = 100$
5) To approximate an "ideal" response from a closed loop control system under Proportional Control:

☐ The closed loop system gain should be as close to zero as possible
☐ The closed loop system gain should be as close to one as possible
☐ The closed loop system gain should be as high as possible without destabilizing the system
☐ The closed loop system gain should be as low as possible without destabilizing the system

PART D (5 marks)

Consider three figures below – each shows responses, to a unit reference signal, of two closed loop control systems. One of the two traces in each of the three figures is a certain second order system with two complex conjugate poles, the same in all three cases. The second trace in each of the three figures is of a system that has the same two complex conjugate poles, plus an additional singularity, i.e. either a pole or a zero.

For each of the three figures, indicate which trace is the second order system with just the two complex poles and then choose one of the four statements shown, as it applies to the figure at hand.

Figure A. The system with just two complex poles is Trace # ____

☐ The other system has two complex poles and a significant real LHP pole
☐ The other system has two complex poles and a significant real LHP zero
☐ The other system has two complex poles and a significant real RHP pole
☐ The other system has two complex poles and a significant real RHP zero
Figure B. The system with just two complex poles is Trace # ____

☐ The other system has two complex poles and a significant real LHP pole
☐ The other system has two complex poles and a significant real LHP zero
☐ The other system has two complex poles and a significant real RHP pole
☐ The other system has two complex poles and a significant real RHP zero

Figure C. The system with just two complex poles is Trace # ____

☐ The other system has two complex poles and a significant real LHP pole
☐ The other system has two complex poles and a significant real LHP zero
☐ The other system has two complex poles and a significant real RHP pole
☐ The other system has two complex poles and a significant real RHP zero
Question 2 (Compulsory)

*Frequency response, Routh-Hurwitz and Bode Criteria of Stability.*

**PART A (6 marks)**

Consider a certain closed loop control system under Proportional Control, as shown. Open loop frequency response plots of the system with $K_p=1$ are shown in Figure Q2.1.

Find the system Gain Margin, Phase Margin and corresponding crossover frequencies. Determine the critical value of the gain, $K_{crit}$, at which the system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, $\omega_{osc}$. Determine the range of gains $K_p$ to provide a stable closed loop system response. Complete Table Q2.1.

**PART B (14 marks)**

Verify the results from Part A by applying the Routh-Hurwitz Criterion of Stability: find the critical value of the gain, $K_{crit}$, at which the system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, $\omega_{osc}$. 
Figure Q2.1

<table>
<thead>
<tr>
<th>Table Q2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Margin in dB and in V/V</td>
</tr>
<tr>
<td>$G_m dB = \omega_{cg} = \Phi_m = \omega_{cp} = K_{crit} = \omega_{osc} =$</td>
</tr>
<tr>
<td>Range of gains for stable closed loop operation</td>
</tr>
</tbody>
</table>
Question 2 Cont'd

Question 3

Second Order Model, Step Response Specifications, System Type.

PART A (10 marks)

Consider a certain closed loop control system under Proportional Control:

1) Assume the Controller gain to be $K_p = 3$, calculate a transfer function of the closed loop system and determine the closed loop system damping ratio, $\zeta$, the closed loop frequency of natural oscillations, $\omega_n$ and the closed loop DC gain, $K_{dc}$. Place your answers in Table Q3.1.

2) Estimate the closed loop system transient and steady state response specifications and place them in Table Q3.2.
Question 3 Cont'd

<table>
<thead>
<tr>
<th>Table Q3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio</td>
</tr>
<tr>
<td>Frequency of natural oscillations</td>
</tr>
<tr>
<td>Closed loop DC gain</td>
</tr>
</tbody>
</table>
**Question 3 Cont'd**

<table>
<thead>
<tr>
<th>$PO =$</th>
<th>$y_{ss} =$</th>
<th>$y_{max} =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{period} =$</td>
<td>$T_{settle \pm 2%} =$</td>
<td>$T_{rise(10-90%)} =$</td>
</tr>
<tr>
<td>System Type is:</td>
<td>$K_{pos} =$</td>
<td>$e_{ss(step)%} =$</td>
</tr>
<tr>
<td>Explain why:</td>
<td>$K_{v} =$</td>
<td>$e_{ss(ramp)} =$</td>
</tr>
<tr>
<td></td>
<td>$K_{a} =$</td>
<td>$e_{ss(parabolic)} =$</td>
</tr>
</tbody>
</table>
PART B (10 marks)
Consider a closed loop response to a unit reference signal of a certain control system. Create a second order model for the system. Calculate the following model parameters: \( \zeta, \omega_n, K_d \) as well as the complete model transfer function \( G_{model}(s) \) and place your answers in Table Q3.3.
**Question 3 Cont'd**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio</td>
<td>( \zeta = )</td>
</tr>
<tr>
<td>Frequency of natural</td>
<td>( \omega_n = )</td>
</tr>
<tr>
<td>oscillations</td>
<td></td>
</tr>
<tr>
<td>Closed loop DC gain</td>
<td>( K_{dc} = )</td>
</tr>
<tr>
<td>Model transfer function</td>
<td>( G_{model}(s) = )</td>
</tr>
</tbody>
</table>
Question 4

*Dominant Poles Second Order Model, Lead Controller Design by Pole Placement, Effect of Additional Zeros on System Response*

Consider a modified control system from the previous question, which is supposed to work under a Lead Controller:

1) **(10 marks)** Find values of the controller parameters, \(a_1, a_0, b_1\) such that: the closed loop system has a dominant pair of complex poles with the frequency of damped oscillations equal to 2 rad/sec, and the corresponding time constant of 1 second, and a third real pole with the corresponding time constant of 0.05 seconds.

2) **(5 marks)** Estimate the transient and steady state response specifications for the compensated system and place them in Table Q.4.1.

3) **(3 marks)** Find the closed loop transfer function of the compensated system, \(G_{cl}(s) = \frac{Y(s)}{R(s)}\).

4) **(2 marks)** Consider step response plots shown on the next page. One of the responses shown is the response of the system compensated according to instructions above, i.e. \(G_{cl}(s)\), while the other one is the response of the system described by the transfer function \(G_1(s)\), described below. Label the two traces on the plot (i.e. Trace A and Trace B), by indicating which trace corresponds to which transfer function. Provide a brief explanation of why that is.

\[G_1(s) = \frac{100}{(s + 20)(s^2 + 2s + 5)}\]
Question 4 Cont'd

<table>
<thead>
<tr>
<th>PO =</th>
<th>$T_{period}$ =</th>
<th>$T_{rise(10-90%)}$ =</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{settle\pm 2%}$ =</td>
<td>$K_{pos}$ =</td>
<td>$e_{ss(step)%}$ =</td>
</tr>
</tbody>
</table>

System Type is:

| $K_v$ = | $e_{ss(ramp)}$ = |

Explain why:

| $K_a$ = | $e_{ss(parabolic)}$ = |
Question 4 Cont'd
Question 5

*Root Locus Analysis, Second Order Dominant Poles Model, Step Response Specifications.*

Consider a certain closed loop control system under Proportional Control:

![Control System Diagram]

1) **(12 marks)** In the space provided in Figure Q5.1, sketch a detailed Root Locus for the system, including crossovers with the imaginary axis, if any, break-away/break-in coordinates, if any, asymptotes, if any, centroid, etc. If you are using estimates, explain why. Place the Root Locus parameters in Table Q5.1.

2) **(4 marks)** Determine the value of the gain $K_p$ that would result in the closed loop system equivalent damping ratio of $\zeta = 0.707$. What is the system Gain Margin at this value of the gain? Place your answers in Table Q5.2.

3) **(4 marks)** Find the closed loop transfer function at the gain as calculated. Would the system closed loop behaviour be well approximated by a second order model at this gain setting? Briefly justify your answer. If it is yes, determine the remaining model parameters $K_{dc}$ and $\omega_n$, and write the model transfer function $G_m(s)$. What will be the expected Percent Overshoot, Settling Time and Steady State Error of the closed loop step response? Place your answers in Table Q5.3.

| Table Q5.1 |
|-------------------|-----|
| Root Locus centroid is at: | $\sigma =$ |
| Root Locus asymptotic angles are equal to: | $\theta_i =$ |
| The critical value of the Proportional Gain for which the system is marginally stable, and the frequency of resulting oscillations: | $K_{cri} =$ |
| $\omega_{osc} =$ |
| Break-away (leave blank if not applicable) coordinate is at: | $s_{ba} =$ |
| Break-in (leave blank if not applicable) coordinate is at: | $s_{bi} =$ |
Question 5 Cont'd
Question 5 Cont'd
Question 5 Cont'd

<table>
<thead>
<tr>
<th>Controller gain required for $\zeta = 0.707$</th>
<th>$K_p =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Margin for this value of the gain</td>
<td>$G_m =$</td>
</tr>
</tbody>
</table>
Question 5 Cont'd

<table>
<thead>
<tr>
<th>Closed loop transfer function</th>
<th>( G_{\text{closed}}(s) = )</th>
<th>Estimates of the closed loop step response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio</td>
<td>( \zeta = )</td>
<td>Percent Overshoot</td>
</tr>
<tr>
<td>Frequency of natural oscillations</td>
<td>( \omega_n = )</td>
<td>Settling Time (within ( \pm 2% ) of steady state)</td>
</tr>
<tr>
<td>Closed loop DC gain</td>
<td>( K_{dc} = )</td>
<td></td>
</tr>
<tr>
<td>Model transfer function</td>
<td>( G_{\text{model}}(s) = )</td>
<td>Steady State Error in ( % )</td>
</tr>
</tbody>
</table>
Question 6

Second Order Dominant Poles Model from Frequency Response Plots, Step Response Specifications.

Consider a certain unit feedback closed loop system under Proportional Control where the process transfer function \( G(s) \) is known as:

\[
G(s) = \frac{100}{s^3 + 12.5s^2 + 26s + 10}
\]

1) **(14 marks)** Frequency response plots of the open loop (note magnitude in dB) and closed loop (note magnitude in V/V), assuming Controller Gain \( K_{op} = 1 \), are shown below and on the next page, respectively. Use the information contained in the frequency response plots to derive two second order approximate models for the closed loop system, one from the open loop Bode plots and the other from the closed loop magnitude Bode plot and complete Table Q6.1.

2) **(7 marks)** Which model do you think will be more accurate? Briefly explain why. Based on your choice, use that model to estimate time response specifications and complete Table Q6.2.
Table Q6.1

<table>
<thead>
<tr>
<th>System Model from Open Loop Bode Plots</th>
<th>System Model from Closed Loop Bode Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta = )</td>
<td>( \zeta = )</td>
</tr>
<tr>
<td>( \omega_n = )</td>
<td>( \omega_n = )</td>
</tr>
<tr>
<td>( K_{dc} = )</td>
<td>( K_{dc} = )</td>
</tr>
<tr>
<td>( G_{m1}(s) = )</td>
<td>( G_{m2}(s) = )</td>
</tr>
</tbody>
</table>
Table Q6.2

<table>
<thead>
<tr>
<th>PO =</th>
<th>$T_{period}$ =</th>
<th>$T_{rise(10-90%)}$ =</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{settle\pm2%}$ =</td>
<td>$K_{pos}$ =</td>
<td>$e_{ss(step)}%$ =</td>
</tr>
<tr>
<td>System Type is:</td>
<td>$K_{V}$ =</td>
<td>$e_{ss(ramp)}$ =</td>
</tr>
<tr>
<td>Explain why:</td>
<td>$K_{a}$ =</td>
<td>$e_{ss(parabolic)}$ =</td>
</tr>
</tbody>
</table>
Question 7

Controller Design in Frequency Domain, Second Order Dominant Poles Model from Frequency Response Plots, Step Response Specifications.

Consider a certain unit feedback closed loop system under Proportional Control where the process is described by the following transfer function:

\[ G(s) = \frac{100}{(s + 1)^2(s + 10)} \]

Frequency response plots of \( G(s) \) are shown in Figure Q7.1.

![Bode Diagram](image)

Figure Q7.1

1) **(5 marks)** Estimate the transient and steady state specifications of the uncompensated closed loop system response and place them in Table Q7.1. Clearly show how you arrived at the estimates of these specifications.
2) **(10 marks)** The system is to be compensated with a Lag Controller with a transfer function as described:

\[ G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1} \quad b_1 > \frac{a_1}{a_0} \quad \text{or} \quad G_c(s) = K_{dc} \frac{\alpha s + 1}{\tau s + 1} \quad \alpha < 1 \]

Design the Lag Controller such that the following requirements are met:
- Steady State Error \( e_{ss \text{step}} \) for the compensated closed loop system is less than 2%
- Phase Margin \( \Phi_m \) for the compensated system is equal to 50°

Clearly specify controller parameters in either of the forms shown and superimpose the compensated system plots on top of the plots in Figure Q7.1 to illustrate your design.

3) **(5 marks)** Estimate the transient and steady state specifications of the compensated closed loop system response and place them in Table Q7.1. Clearly show how you arrived at the estimates of these specifications.
Question 7 Cont'd

<table>
<thead>
<tr>
<th>Uncompensated System</th>
<th>Compensated System</th>
<th>Uncompensated System</th>
<th>Compensated System</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PO =$</td>
<td>$PO =$</td>
<td>$K_v =$</td>
<td>$K_v =$</td>
</tr>
<tr>
<td>$T_{settle \pm 2%} =$</td>
<td>$T_{settle \pm 2%} =$</td>
<td>$e_{ss(ramp)} =$</td>
<td>$e_{ss(ramp)} =$</td>
</tr>
<tr>
<td>System Type is:</td>
<td>System Type is:</td>
<td>$K_a =$</td>
<td>$K_a =$</td>
</tr>
<tr>
<td>$K_{pos} =$</td>
<td>$K_{pos} =$</td>
<td>$e_{ss(parabolic)} =$</td>
<td>$e_{ss(parabolic)} =$</td>
</tr>
<tr>
<td>$e_{ss(step)}% =$</td>
<td>$e_{ss(step)}% =$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Question 7 Cont'd
Question 8

State Space Model, Canonical Forms, Eigenvalues, Transfer Function from State Space, Controllability and Observability, Pole Placement by State Feedback Method.

Consider a control system described by the following state space model:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & -4 & -4 & -1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix} u
\]

\[y = \begin{bmatrix}
1 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
\]

1) **(13 marks)** For the system in question do the following:

a) find the system transfer function, \( G(s) = \frac{Y(s)}{U(s)} \)

b) find the system eigenvalues

c) check if the system is controllable and if it is observable.

2) **(7 marks)** Assume the state feedback is of the form:

\[u = K \cdot \mathbf{x} + r \quad \text{where: } K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}\]

and use pole placement by state feedback to place the compensated system eigenvalues at: -3, -4, -5 and -6. What are the gain values in the vector \(K\)?

**HINT:** Avoid complicated matrix manipulations in derivation of results and use canonical properties of the system instead.
Question 8 Cont'd