National Exams December 2011
04-BS-1, Mathematics
3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:

1. (a) 12 marks, (b) 8 marks
2. (a) 10 marks, (b) 10 marks
3. 20 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. (a) 3 marks, (b) 3 marks, (c) 14 marks
8. 20 marks
1. Let $P$ be the plane passing through the three points $(0,1,2)$, $(1,3,-1)$ and $(2,0,1)$.
   (a) Find an equation representing the plane $P$.
   (b) Find the line of intersection between the plane $P$ and the plane $x - 2y + z = 3$.

2. (a) Find the eigenvalues and the eigenvectors of the matrix

$$
\begin{pmatrix}
3 & -2 \\
1 & 1
\end{pmatrix}
$$

(b) Solve the system of differential equations

$$
\frac{dx}{dt} = 3x - 2y, \\
\frac{dy}{dt} = x + y + e^{-2t}.
$$

subject to the initial conditions $x(0) = 2$, $y(0) = -1$.

3. Find the solution, $y(x)$, of the differential equation

$$
y'' + 9y = \sec 3x,
$$

$y'(0) = 0$, $y(0) = 1$. Note that $'$ denotes differentiation with respect to $x$.

4. Find an equation for the line tangent to the intersection of the surfaces

$$
x^2 + y^2 - 6z = 11
$$

and

$$
4x^2 + y^2 + z^2 - 4y - 4z + 3 = 0
$$

at the point $(1,0,2)$.
5. At what angle does the line represented parametrically by $z = 2 - t$, $y = t$, $x = 2 + 2t$ intersect the hyperboloid $z = 4 - x^2 + y^2$? You may leave your answer as an inverse sine or cosine.

6. Let $S$ be the surface of the region defined by $x^2 + 4y^2 \leq 1$, $x \geq 0$, $y \geq 0$, $0 \leq z \leq 4$, and let $F$ be the vector function $F(x, y, z) = (y^3, x^3, z^3)$. Evaluate the integral of $F$ over the surface $S$.

7. Let $C$ be the curve formed by the intersection of the cylinder $x^2 + y^2 = 9$ and the plane $z = 1 + y - 2x$, travelled clockwise as viewed from the positive $z$-axis, and let $v$ be the vector function $v = 4xi - 2yj + 2yk$.

(a) Evaluate the divergence of $v$

(b) Evaluate the curl of $v$

(c) Evaluate the line integral $\int_C v \cdot dr$.

8. Find the general solution of the differential equation

$$x^2y'' - 4xy' + 6y = 3x^4.$$  

Note that $'$ denotes differentiation with respect to $x$. 