National Exams May 2011

04-Chem-A6, Process Dynamics & Control

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is an OPEN BOOK EXAM.
   Any non-communicating calculator is permitted.

3. FIVE (5) questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked.

4. Each question is of equal value.

5. Most questions require an answer in essay format. Clarity and organization of the answer are important.
Note 1: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

**Problem #1** (20% total)

Consider a closed loop system composed of the following elements:

1 - a proportional controller with gain $K_c$,

2 - a process transfer function $G_p$,

3 - a sensor transfer function $H$.

\[
G_p = \frac{1}{(s+1)^3}
\]

Find the maximum $K_c$ for the following 2 cases:

(10%)

(a) $H = 1$

(10%)

(b) $H = e^{-0.7s}$

If iterations are required to solve an equation, show only the first 3 iterations (steps).

**Problem #2** (20% total)

Two stirred tank reactors are separated by a plug-flow pipe with an associated dead time of $D$ seconds as shown in the figure.

Assume constant holdups $V_1$ and $V_2$, constant mass flow $F$, constant density, isothermal operation at temperatures $T_1$ and $T_2$ and first order kinetics with simultaneous reactions

\[
A \xrightarrow{k_1} C \quad \text{and} \quad A \xrightarrow{k_2} C.
\]

No reaction occurs in the plug flow section.

(10%)

1 - Develop the modelling equations describing the system.
2- Find the transfer function:

\[
\frac{C_{A_2}(s)}{C_{A_0}(s)}
\]

**Problem #3 (20% total)**

For the draining tank shown in the figure

Compute the change in level \( \delta h(t) \) as a function of time for the following two cases:

1. a step of one unit in inlet flow \( F_0 \)
2. a unit impulse in inlet flow \( F_0 \)

The cross-section area of the tank is 1 m\(^2\). Initial level = 7 m

The flow out is given by \( F_1 = R_1 \cdot h \), where the hydraulic resistance \( R_1 = \frac{2m^2}{\text{min}} \).
Problem #4 (20% total)

A process is described by the following transfer function

\[ G_p = \frac{4(2s + 1)e^{-2s}}{s(5s + 1)} \]

(10%) 1 - Compute the output response of this system to a unit step in the input.

(10%) 2 - Compute expressions for the amplitude ratio and phase angle for a sinusoidal input as functions of frequency.

Problem #5 (20% total)

A process is described by the following transfer function

\[ G_p = \frac{10e^{-5s}}{100s + 1} \]

(a) Design an IMC (Internal Model Controller) for this process. Show your design with a block diagram. DO NOT USE PADE APPROXIMATION.

(b) Compute and plot the closed loop response to a unit step change in the set point using the controller obtained in (a). Select the time constant for the IMC filter to be \( \tau_c = 10 \). Assume perfect model (no model error).

(c) If the time delay is approximated by using a 1-1 Pade approximation calculate the PID tuning parameters of the feedback controller equivalent to the IMC design.
**Problem #6** (20% total)

For the process modelled by:

\[
\frac{dy_1}{dt} = -y_1 - y_2 + x_1 \\
\frac{dy_2}{dt} = y_1 - 2y_2 + x_1 + x_2
\]

(10%) (a) Find the four transfer functions relating the inputs \(x_1\) and \(x_2\) to the outputs \(y_1\) and \(y_2\). The x's and y's are deviation variables.

(10%) (b) Compute \(y_2\) as a function of time for \(x_1 = 0\) and a unit step in \(x_2\).

**Problem #7** (20% total)

A process given by:

\[
G_p = \frac{e^{-0.1s}}{0.5s + 1}
\]

is controlled by a proportional controller with gain \(K_c\).

(10%) (a) Plot qualitatively the Bode Plot for this system (show slope values, corner frequencies and extreme amplitude and phase values).

(10%) (b) Compute the gain \(K_c\) to obtain a gain margin of 1.7.

**Problem #8** (20% total)

A process given by:

\[
G_p = \frac{20}{s - 3}
\]

is controlled by a proportional controller with gain \(k_c\).

(10%) (a) Show a qualitative Nyquist Plot (show only 3-4 points and the general shape of the plot for this problem) for \(K_c = 1\). Is the system closed loop stable for \(K_c = 1\)?

(10%) (b) Based on the Nyquist criterion, compute a range of \(K_c\) values to obtain closed loop stability.