PROFESSIONAL ENGINEERS ONTARIO

National Examinations - May 2011

07-Mec-A5, Electrical & Electronics Engineering

Mechanical Engineering

3 hours duration

Name [print]:

Signature:

Notes:

[1] If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

[2] Candidates may use one of two calculators, the Casio or Sharp approved models. This is a closed book examination.

[3] Any five (5) questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.


[5] Clarity and organization of answers are important.

[6] The candidate is required to sign this examination paper and submit it with the solution booklets.

[7] \( \pi = 3.14159 \)

\( 1 \text{ hp} = 745 \text{ W} \)

\( \mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1} \)
QUESTION 1

Consider the circuit shown in Figure 1 which has been designed using ideal operational amplifiers (U₁ to U₃) with infinite bandwidth and infinite open loop gain. In the schematic, a, b and c are constants. You will note that U₃ is configured as a basic difference amplifier which has a transfer function given by:

\[ E_0 = c \left( e_y - e_x \right) \]

where \( e_y \) and \( e_x \) are the potentials at points y and x respectively.

In the derivation of the transfer function for such circuits, one can assume:

[i] Zero differential voltage between the input terminals of the operational amplifier,
[ii] Zero current flows into either input terminal of the operational amplifier.

Applying the principle of superposition, derive an expression for the transfer function of the total circuit [\( E_0 \) as a function of \( E_1, E_2 \)].

**Hint:** Let \( E_2 = 0 \), and solve for the potentials at points x and y for input \( E_1 \).
Let \( E_1 = 0 \), and again solve for the potentials at points x and y for input \( E_2 \).
Calculate the resultant output \( E_0 \) for both \( E_1 \) and \( E_2 \) inputs.

![Figure 1 Circuit Schematic](image-url)
QUESTION 2

Consider the transistor circuit shown in Figure 2. All transistors can be assumed to be identical with a dc current gain $\beta$.

Calculate the current transfer ratio for the circuit, $I_2 / I_1$, as a function of $\beta$.

![Figure 2  Transistor Circuit](image-url)
QUESTION 3

A novel type of DC machine is designed using a disc type rotor of effective outer and inner diameters D and d respectively, as shown in Figure 3. A current $I_2$ is fed radially through the rotor via two ring shaped carbon brushes. The rotor lies in the horizontal plane and is situated in a vertical magnetic field of uniform density, B Tesla. The rotor spins at an angular speed $\omega$ rad/s.

[a] Find the magnitude of the emf $e$ generated between the brushes.

[b] Determine the torque that the rotor will be subjected to and find the output horsepower of the machine.

HINT: As a starting point, consider an elemental annulus of radius $r$ and radial length $dr$. 

![Figure 3 dc Machine Diagram]
QUESTION 4

Consider the magnetic circuit of a transformer shown in Figure 4. Infinite relative permeability can be assumed for the iron core.

![Transformer Diagram]

**Figure 4 Transformer**

The following specifications apply.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( L_1 )</td>
<td>( 3.77 \times 10^{-2} \text{ m} )</td>
<td>( A_1 )</td>
<td>( 0.02 \text{ m}^2 )</td>
</tr>
<tr>
<td>( L_2 )</td>
<td>( 7.54 \times 10^{-2} \text{ m} )</td>
<td>( A_2 )</td>
<td>( 0.02 \text{ m}^2 )</td>
</tr>
<tr>
<td>( N_1 ) [primary]</td>
<td>200 turns</td>
<td>( N_2 ) [secondary]</td>
<td>20 turns</td>
</tr>
</tbody>
</table>

When a dc voltage equal to 10 mV is applied to the primary, the measured primary current is 100 mA. When a dc voltage of 0.1 mV is applied to the secondary winding, the measured secondary current is 100 mA.

Assume that leakage inductances and eddy current and hysteresis losses are negligible; consider an operating frequency of 1000 Hz.

[a] Draw the equivalent circuit of the transformer referred to the primary and calculate component values.

[b] A transducer with an impedance of 0.078 Ω is connected across the secondary of the transformer; an amplifier is connected to the primary. Calculate the output impedance of the amplifier to give maximum power transfer to the load.
QUESTION 5

This question consists of two parts which are not necessarily related.

Part I
A combinational logic circuit is shown in Figure 5.
[a] Write a general Boolean algebra expression for the output F as a function of the inputs A, B.
[b] Using DeMorgan’s theorems and other Boolean identities, simplify the expression obtained in [a]. Is there a single gate which can replace the network shown?
[c] Generate a truth table giving the logic levels at points C, D, E and F for inputs A,B.

Part II
Design a 2-input exclusive or (EOR) gate using only 2-input NOR gates.
[d] Develop the truth table for the gate.
[e] Write a general Boolean algebra expression for the output as a function of the inputs.
[f] Using DeMorgan’s theorems and other Boolean identities, modify the expression obtained in [e] to provide a solution which can be implemented with NOR gates.
[g] Draw the circuit diagram for the final gate array.

Figure 5 Combinational Logic Circuit
QUESTION 6

Consider the RC circuit shown in Figure 6[a]. The switch $S_1$ is closed at time $t=0$ connecting the dc supply $V_1$ to the network.

[a] Derive an expression for the transfer function of the circuit, $V_o/V_i$, in the time domain.

[b] Sketch the transfer function for a time interval of 5 time constants.

The RC circuit is reconfigured as shown in Figure 6[b]. An ac voltage source of variable frequency $v_i$ is connected to the input.

[c] Derive an expression for the transfer function of the circuit, $v_o/v_i$, in the frequency domain.

[d] Sketch the magnitude of the transfer function for a frequency range of 4 decades centered at the corner frequency of the circuit.

Figure 6 RC Circuit: [a] dc test; [b] ac test
QUESTION 7
This question consists of two parts which are not necessarily related.

Part I
A 3 phase, 300 hp, 12 pole wound rotor induction motor is operated from a 60 Hz source. The per phase rotor resistance $r_2$ was measured and found to be 0.04Ω. At full load, the speed of the motor is 582 rpm.

At full load, determine:

[a] The speed of the magnetic field in revolutions per minute.
[b] The slip of the rotor.
[c] The frequency of the rotor currents.
[d] The angular velocity of the stator field with respect to the stator.
[e] The angular velocity of the stator field with respect to the rotor.
[f] The angular velocity of the rotor field with respect to the rotor.
[g] The angular velocity of the rotor field with respect to the stator.

Part II
In the normal operating region of an induction motor, torque is a linear function of slip. A test was performed on a 3 phase, 8 pole squirrel cage induction motor which is operated from a 60 Hz source and it was found that it developed a torque of 3 N.m at a speed of 810 rpm.

The induction motor is used to drive a load which requires a torque which is a linear function of speed. In another test, it was found that the torque required by the load was 0.5 N.m at a speed of 435 rpm.

[a] Sketch the speed-torque characteristics for the motor and load.
[b] Calculate the operating point for the motor-load system.
QUESTION 8

An industrial load is represented in Figure 8 by \( R = 6\Omega \) and \( X_L = 8\Omega \). The load voltage is \( 250\angle 0^\circ \) V.

[a] Calculate the load current, power, reactive power and power factor.

[b] Calculate the generator voltage \( V_G \) required at the input end of the transmission line (represented by the series impedance \( Z_T = (1 + j3)\Omega \)) and the power lost in transmission \( P_T \).

[c] If capacitor \( X_C = 12.5\Omega \) is connected in parallel by closing switch \( S \), calculate \( I_C \), the new load current \( I \), and the new power factor. Show \( V, I_L, I_C \), and \( I \) on a phasor diagram.

[d] Calculate the new generator voltage and the new transmission power loss.

[e] What two advantages do you see for improving the power factor by adding a parallel capacitor?

![Figure 8 Industrial Load](image)