NATIONAL EXAMINATIONS
December 2012
07-MEC-A6-1 FLUID MACHINERY
Three hours duration

Notes to Candidates

1. This is a Closed Book examination.

2. Exam consists of two Sections Section A is Calculative (5 questions) and Section B is Descriptive (3 questions).

3. Do four (4) questions (including all parts of each question) from Section A (Calculative) and two (2) questions from Section B (Descriptive). Note also that Question 6 and Question 7 are on the same page.

4. Descriptive questions require answers of approximately two pages each, including pages provided, with complete explanations and sketches, if appropriate, to support the explanation.

5. Six questions constitute a complete paper. (Total 60 marks).

6. All questions are of equal value. (Each 10 marks).

7. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

8. If any initial parts of a multi-part question cannot be solved the remaining parts may be worked by making appropriate assumptions from the technical data given.

9. Candidates may use one of the approved Casio or Sharp calculators.

10. Reference data for particular questions are given in the Attachments on pages 9 to 13. All pages on which answers have been written are to be returned with the answer booklet. Candidates names must be on these sheets.

11. Reference formulae and constants are given on pages 13 to 17.

12. Drawing Instruments (scale ruler, protractor and sharp pencil) are required for vector diagrams. Alternatively the questions may be solved mathematically.
SECTION A  CALCULATIVE QUESTIONS

QUESTION 1  COMPRESSORS AND TURBINES

Consider a simple single shaft gas turbine power plant having the following specifications:

Compressor pressure ratio 12.0
Ambient pressure 100 kPa
Ambient temperature 15°C (288°K)
Turbine inlet temperature 1077°C (1350°K)
Compressor efficiency 0.86
Turbine efficiency 0.89
Mechanical (shaft) efficiency 0.98
Air/fuel ratio 50

(a) Sketch a T-s diagram showing the complete thermodynamic cycle and identify by number all key points in the cycle.

(b) Determine and identify by number the temperatures at all key points in the cycle.

(c) Calculate the specific work output from the unit in kWs/kg of gas flow.

(d) Calculate the specific fuel consumption of the unit in kg/kWh

[ 10 marks ]
QUESTION 2  COMPRESSOR FIRST STAGE

Refer to the Examination Paper Attachments Page 9 Acacia and Port Rex Power Stations and Page 10 Compressor Velocity Diagram.

Each power station has three units. Each unit has an output of 60 MW and is powered by twin back to back gas turbines driving a common electrical generator. The diagram and specifications on Page 9 are for one gas turbine only.

Consider the first stage of the compressor (N1 rotor) which has the following approximate parameters:

- Rotor hub diameter at inlet \( D_1 = 480 \text{ mm} \)
- Blade tip diameter at inlet \( D_2 = 1120 \text{ mm} \)
- Inlet guide vane outlet angle \( \alpha_1 = 30^\circ \)
- First stage moving blade outlet angle \( \beta_2 = 40^\circ \)

Assume that the plant is operating under the following conditions:

- Rotational speed \( N = 6800 \text{ rev/min} \)
- Air flow at inlet to compressor \( M = 136 \text{ kg/s} \)
- Air temperature at inlet to compressor \( T_1 = 15^\circ \text{C} \) \((288^\circ \text{K})\)

Assume that ideal conditions prevail (no friction losses).

(a) Calculate the blade velocity \( U \) and air inlet axial velocity \( C_{x1} \).

(b) Draw to scale (see note below) the velocity diagrams at the first stage moving blade inlet and outlet and measure the absolute velocities \( C_1 \) and \( C_2 \) and relative velocities \( W_1 \) and \( W_2 \).

(c) Determine the work done in kJ/kg and power input to the first stage in kW.

(d) Determine the enthalpy rise and hence temperature rise in the first stage.

(e) Determine the pressure ratio of the first stage assuming isentropic conditions.

Note: The scale drawing should be to a large enough scale for accurate measurements (a scale of 10 m/s = 4 mm is suggested). While calculation of velocities by trigonometric ratios is acceptable it is longer and more time consuming.

[10 marks]
QUESTION 3  MULTI-JET PELTON TURBINE

Consider the design of a multi-jet Pelton wheel with parameters and operating conditions as given below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>$H = 200,\text{m}$</td>
</tr>
<tr>
<td>Flow rate</td>
<td>$Q = 4,\text{m}^3/\text{s}$</td>
</tr>
<tr>
<td>Nozzle velocity coefficient</td>
<td>$K = 0.99$</td>
</tr>
<tr>
<td>Wheel diameter</td>
<td>$D = 1.47,\text{m}$</td>
</tr>
<tr>
<td>Mechanical efficiency</td>
<td>$\eta = 88%$</td>
</tr>
</tbody>
</table>

The following conditions are desirable:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade speed to jet speed ratio</td>
<td>0.47</td>
</tr>
<tr>
<td>Jet diameter to wheel diameter ratio</td>
<td>0.113</td>
</tr>
</tbody>
</table>

(a) Calculate the wheel rotational speed.

(b) Calculate the power output.

(c) Determine the number of nozzles required

(d) Calculate the specific speed of the machine

[ 10 marks ]
QUESTION 4 HYDRO TURBINE MODEL

Technical specifications for the hydro turbines at Vanderkloof Hydro Power Station are as follows:

- Generator design output: 120 MW (at 0.90 power factor lagging)
- Speed of machine: 125 rev/min
- Electrical frequency: 50 Hz
- Generator voltage: 11 kV
- Design net head: 65 m
- Design water flow: 200 m$^3$/s
- Maximum water flow: 213 m$^3$/s
- Turbine runner diameter: 5.462 m
- Turbine runner material: stainless cast steel

(a) Calculate the specific speed of the machine.

(b) Calculate the overall efficiency of the turbine based on the design parameters.

In practice the actual efficiency is not measured directly on site due to the difficulty and cost in measuring large volume flows with sufficient accuracy. A model test is therefore carried out at the manufacturer's works to prove the performance of the prototype machine. Assume that an homologous (scaled to be geometrically identical) model runner 200 mm in diameter is available and can be tested in an instrumented hydraulic system under a head of 10 m. Use the turbine affinity laws or similarity rules to answer the following:

(c) Determine the speed at which the model should run.

(d) Determine the necessary flow through the model.

(e) Determine the ideal (no friction) power developed by the model.

The efficiency of the model and the prototype are not identical due to different hydraulic friction values arising due to scaling. The Moody equation allows the hydraulic efficiencies of the model and prototype to be compared.

(f) Assuming that the prototype has an electrical efficiency of 98%, determine the efficiency that should be measured on the model to ensure that the prototype will meet its specified efficiency.

[ 10 marks ]
QUESTION 5  PUMP PERFORMANCE

Refer to the Examination Paper Attachments Page 11 Pump Velocity Diagram

The attached diagram clarifies the nomenclature to be used in answering the question.

The picture below the velocity diagram shows a pump impeller of a radial flow centrifugal pump for pumping water. The key dimensions are as follows:

- Blade inner diameter \( D_1 = 130 \text{ mm} \)
- Blade outer diameter \( D_2 = 300 \text{ mm} \)
- Blade inner height \( h_1 = 20 \text{ mm} \) (in axial direction)
- Blade outer height \( h_2 = 10 \text{ mm} \) (in axial direction)
- Blade inlet angle \( \beta_1 = 20^\circ \)
- Blade outlet angle \( \beta_2 = 25^\circ \)

- Pump speed \( N = 1750 \text{ rev/min} \)
- Water flow rate \( Q = 0.030 \text{ m}^3/\text{s} \)
- Hydraulic head \( H = 35 \text{ m} \)

For the given speed and a flow rate draw to scale (see note below) the velocity diagrams at inlet and outlet and determine the following neglecting the vane thickness:

(a) Tangential blade velocities at inlet and outlet
(b) Radial water velocity at inlet and outlet
(c) Tangential water velocity at inlet and outlet
(d) Torque and power required to drive the impeller
(e) Hydraulic power and efficiency of pump

Note: The scale drawing should be to a large enough scale for accurate measurements (a scale of 1 cm = 2 m/s is suggested). While calculation of velocities by trigonometric ratios is acceptable it is longer and more time consuming.

[ 10 marks ]
SECTION B  DESCRIPTIVE QUESTIONS

Note that each ten mark question requires an answer of approximately two full pages, including pages provided, with complete explanations with sketches, if appropriate, to support the explanation.

QUESTION 6  NUMBER OF STAGES

Refer to the Examination Paper Attachments Page 9 Acacia and Port Rex Power Stations.

The cross section of the gas turbines shows the number of stages in the compressor and the number of stages in the turbine. Explain why the compressor has many more stages than the turbine. Give the reasons for selecting the appropriate number of stages for both the compressor and the turbine. In particular explain what requires the compressor to have a certain minimum number of stages and why the turbine must have multiple stages.

[ 10 marks ]

QUESTION 7  FAN BLADE SHAPE

Refer to the Examination Paper Attachments Page 12 Fan Characteristics.

Radial flow centrifugal fans may have blades that are radial or curved forwards or backwards. Explain what effect the shape of the blades has on the performance of the fan.

(a)  Show graphically in sketches on Page 12 how the velocity diagram changes and how the head versus flow curve is different for the three conditions.

(b)  Explain the advantages of forward or backward curved blades with respect to the other. Hence clarify with reasons which configuration is more common.

[ 10 marks ]
QUESTION 8  PUMP AND SYSTEM CHARACTERISTICS

Refer to the Examination Paper Attachments Page 13 Pump Characteristics.

(a) On the axes given sketch the following:
   
   (i) Head versus flow characteristics of one centrifugal pump supplying water to a typical pipe system. Identify the operating point.

   (ii) Head versus flow characteristic of a second identical pump in parallel supplying the same system. Identify the operating point.

   (iii) Head versus flow characteristic when a control valve in the system is closed to obtain the same flow as when one pump is operating. Identify the operating point.

(b) If each individual pump and the system without throttling (as in case (i) above) can be mathematically modelled by the following equations

\[ H_{\text{pump}} = K_1 - K_2 Q - K_3 Q^2 \]
\[ H_{\text{system}} = K_4 Q^2 \]

write new equations for cases (ii) and (iii) by inserting appropriate numerical values but keeping the same constants which will allow the operating points to be determined.

(c) Show graphically and explain what effect a change in speed of the pump in case (i) above would have on the operating point of the system.

[ 10 marks ]
EXAMINATION PAPER ATTACHMENTS

QUESTION 2 & QUESTION 6  ACACIA AND PORT REX POWER STATIONS

Technical Specifications

<table>
<thead>
<tr>
<th></th>
<th>Peak Load</th>
<th>Base Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Output (kW)</td>
<td>60 860</td>
<td>57 100</td>
</tr>
<tr>
<td>Heat Rate (kJ/kWh)</td>
<td>11 791</td>
<td>11 887</td>
</tr>
<tr>
<td>Speed - N1 Rotor (rev/min)</td>
<td>6 805</td>
<td>6 640</td>
</tr>
<tr>
<td>Speed - N2 Rotor (rev/min)</td>
<td>8 395</td>
<td>8 320</td>
</tr>
<tr>
<td>Speed - Power Turbine (rev/min)</td>
<td>3 000</td>
<td>3 000</td>
</tr>
<tr>
<td>Temperature - Gas Generator Turbine Inlet (°C)</td>
<td>1 077</td>
<td>1 043</td>
</tr>
<tr>
<td>Temperature - Power Turbine Inlet (°C)</td>
<td>682</td>
<td>657</td>
</tr>
<tr>
<td>Temperature - Power Turbine Exhaust (°C)</td>
<td>483</td>
<td>467</td>
</tr>
<tr>
<td>Exhaust Gas Flow Rate (kg/s)</td>
<td>278</td>
<td>272</td>
</tr>
<tr>
<td>Gas Generator Pressure Ratio</td>
<td>14.1</td>
<td>13.6</td>
</tr>
</tbody>
</table>

N1  Low Speed Compressor and Turbine
N2  High Speed Compressor and Turbine

Inlet Air Conditions  15°C
QUESTION 2  COMPRESSOR VELOCITY DIAGRAM

Rotor Blade Row

Stator Blade Row

\[
\begin{align*}
U & \quad \text{Blade velocity} \\
C_1 & \quad \text{Rotor blade absolute inlet velocity} \\
W_1 & \quad \text{Rotor blade relative inlet velocity} \\
C_2 & \quad \text{Rotor blade absolute outlet velocity} \\
W_2 & \quad \text{Rotor blade relative outlet velocity} \\
C_3 & \quad \text{Stator blade absolute outlet velocity}
\end{align*}
\]
QUESTION 5  PUMP VELOCITY DIAGRAM

Nomenclature for velocity vectors and angles

\[ V_1 \] Absolute water velocity at inlet
\[ V_{B1} \] Blade velocity at inlet
\[ V_{1R} \] Radial water velocity at inlet
\[ V_{1T} \] Tangential water velocity at inlet
\[ V_2 \] Absolute water velocity at outlet
\[ V_{B1} \] Blade velocity at outlet
\[ V_{2R} \] Radial water velocity at outlet
\[ V_{2T} \] Tangential water velocity at outlet
(a) Velocity Diagrams

Draw velocity diagrams above each diagram and in line with the topmost blade for the three cases given.

(a) Head versus Flow

On the axes given draw the head versus flow characteristics for the three cases above. Label the curves accordingly.
QUESTION 8  PUMP AND SYSTEM CHARACTERISTICS

(a) Head versus Flow.

On the axes below plot head versus flow characteristics for cases (i), (ii) and (iii).

(b) Speed Change

On the axes given show the effect of speed change for case (i) and explain the reason for change in the answer booklet.
**NOMENCLATURE FOR REFERENCE EQUATIONS (SI UNITS)**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Flow area, Surface area</td>
<td>m²</td>
</tr>
<tr>
<td>c_p</td>
<td>Specific heat at constant pressure</td>
<td>J/kg°C</td>
</tr>
<tr>
<td>c_v</td>
<td>Specific heat at constant volume</td>
<td>J/kg°C</td>
</tr>
<tr>
<td>b</td>
<td>Width</td>
<td>m</td>
</tr>
<tr>
<td>C</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>D</td>
<td>Diameter</td>
<td>m</td>
</tr>
<tr>
<td>E</td>
<td>Energy</td>
<td>J</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
<td>N</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>h</td>
<td>Specific enthalpy</td>
<td>J/kg</td>
</tr>
<tr>
<td>h_L</td>
<td>System head</td>
<td>m</td>
</tr>
<tr>
<td>H</td>
<td>Head loss</td>
<td>m</td>
</tr>
<tr>
<td>k</td>
<td>Pump or turbine head</td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>Ratio of specific heats</td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>Length</td>
<td>m</td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
<td>kg</td>
</tr>
<tr>
<td>m</td>
<td>Mass flow rate</td>
<td>kg/s</td>
</tr>
<tr>
<td>N</td>
<td>Rotational speed</td>
<td>rev/s</td>
</tr>
<tr>
<td>N_S</td>
<td>Specific Speed</td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>Pressure</td>
<td>Pa (N/m²)</td>
</tr>
<tr>
<td>P</td>
<td>Power</td>
<td>W (J/s)</td>
</tr>
<tr>
<td>q</td>
<td>Heat transferred</td>
<td>J/kg</td>
</tr>
<tr>
<td>Q</td>
<td>Heat</td>
<td>J</td>
</tr>
<tr>
<td>Q</td>
<td>Flow rate</td>
<td>m³/s</td>
</tr>
<tr>
<td>r</td>
<td>Radius</td>
<td>m</td>
</tr>
<tr>
<td>R</td>
<td>Specific gas constant</td>
<td>J/kg K</td>
</tr>
<tr>
<td>s</td>
<td>Entropy</td>
<td>J/kg K</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>u</td>
<td>Specific internal energy</td>
<td>J/kg</td>
</tr>
<tr>
<td>U</td>
<td>Internal Energy</td>
<td>J</td>
</tr>
<tr>
<td>U</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>V</td>
<td>Specific volume</td>
<td>m³/kg</td>
</tr>
<tr>
<td>v</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>W</td>
<td>Specific work</td>
<td>J/kg</td>
</tr>
<tr>
<td>W</td>
<td>Work</td>
<td>J</td>
</tr>
<tr>
<td>W</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>x</td>
<td>Length</td>
<td>m</td>
</tr>
<tr>
<td>z</td>
<td>Elevation</td>
<td>m</td>
</tr>
</tbody>
</table>
\( \alpha \) Pump blade angle
\( \alpha \) Compressor blade angle
\( \beta \) Pump blade angle
\( \beta \) Compressor blade angle
\( \gamma \) Turbine blade angle
\( \varphi \) Turbine blade angle
\( \delta \) Turbine blade angle
\( \eta \) Efficiency
\( \theta \) Nozzle angle
\( \mu \) Dynamic viscosity
\( \nu \) Kinematic viscosity
\( \rho \) Density
\( \sigma_c \) Critical cavitation parameter
\( \tau \) Thrust
\( \tau \) Torque
\( \phi \) Peripheral velocity factor
\( \omega \) Rotational speed
\( \Omega \) Heat transfer rate

**GENERAL CONSTANTS**

*Use unless otherwise specified*

- Acceleration due to gravity: \( g = 9.81 \text{ m/s}^2 \)
- Atmospheric pressure: \( p_{\text{atm}} = 100 \text{ kPa} \)
- Density of water: \( \rho_{\text{water}} = 1000 \text{ kg/m}^3 \)
- Density of air: \( \rho_{\text{air}} = 1.21 \text{ kg/m}^3 \) (at 15\(^\circ\)C)
- Density of air: \( \rho_{\text{air}} = 1.19 \text{ kg/m}^3 \) (at 20\(^\circ\)C)
- Specific heat of air: \( c_p = 1.005 \text{ kJ/kg}^\circ\text{C} \)
- Specific heat of air: \( c_v = 0.718 \text{ kJ/kg}^\circ\text{C} \)
- Specific heat of water: \( c_p = 4.19 \text{ kJ/kg}^\circ\text{C} \)

**GENERAL REFERENCE EQUATIONS**

**Basic Thermodynamics**

- First Law: \( dE = \delta Q - \delta W \)
- Enthalpy: \( h = u + pv \)
- Continuity: \( \rho VA = \text{constant} \)
Potential Energy: \( E_{PE} = mgz \)
Kinetic Energy: \( E_{KE} = \frac{V^2}{2} \)
Internal Energy: \( E_{IN} = U \)
Flow Work: \( w = \Delta(pv) \)
Energy Equation: \( zg + \frac{V^2}{2} + u + pv + \Delta w + \Delta q = \text{constant} \)

**Ideal Gas Relationships**

Gas Law: \( pv = RT \)
Specific Heat at Constant Pressure: \( c_p = \Delta h / \Delta T \)
Specific Heat at Constant Volume: \( c_v = \Delta u / \Delta T \)
Isentropic Relations: \( \frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^k = \left(\frac{T_1}{T_2}\right)^{\gamma(k-1)} \)

**FLUID MACHINERY REFERENCE EQUATIONS**

**Fluid Mechanics**

Pressure: \( p = \rho gh \)
Continuity Equation: \( p_1V_1A_1 = p_2V_2A_2 = M \)
Bernoulli's Equation: \( p_1/\rho g + z_1 + \frac{V_1^2}{2g} = p_2/\rho g + z_2 + \frac{V_2^2}{2g} \)
Momentum Equation: \( F = p_1A_1 - p_2A_2 - \rho VA(V_2 - V_1) \) (one dimensional)

**Steam Turbines**

Nozzle Equation: \( h_1 - h_2 = \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) \)
Work: \( w = [(V_1^{2\text{absolute}} - V_2^{2\text{absolute}}) + (V_2^{2\text{relative}} - V_1^{2\text{relative}})]/2 \)

**Gas Turbines**

State Equation: \( pv = RT \)
Isentropic Equation: \( \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \)
Enthalpy Change: \( h_1 - h_2 = c_p(T_1 - T_2) \) (ideal gas)
Nozzle Equation: \( h_1 - h_2 = \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) \)

**Compressors**

Work: \( W = U(C_{Y_2} - C_{Y_1}) \)
Rotor Enthalpy Change: \( h_1 + \frac{1}{2}C_{Y_1}^2 = h_2 + \frac{1}{2}C_{Y_2}^2 \)
Stator Enthalpy Change: \( h_2 + \frac{1}{2}C_{Y_2}^2 = h_3 + \frac{1}{2}C_{Y_3}^2 \)
Isentropic Equation: \( \frac{T_3}{T_1} = \left(\frac{p_3}{p_1}\right)^{\frac{k-1}{k}} \)
Jet Propulsion

Thrust: \[ \tau = M(V_{jet} - V_{aircraft}) \]
Thrust Power: \[ \tau V_{aircraft} = M(V_{jet} - V_{aircraft})V_{aircraft} \]
Jet Power: \[ P = M(V_{jet}^2 - V_{aircraft}^2) / 2 \]
Propulsion Efficiency: \[ \eta_p = 2V_{aircraft}/(V_{jet} + V_{aircraft}) \]

Wind Turbine

Maximum Ideal Power: \[ P_{max} = 8\rho AV_1^3 / 27 \]

Energy Equation

Pump and Turbine With Friction: \[ p_1/\rho g + z_1 + V_1^2/2g + w_{in}/g = p_2/\rho g + z_2 + V_2^2/2g + w_{out}/g \]

Hydraulic Machines

Similarity Equations: \[ \frac{Q_M}{Q_P} = \left(\frac{\omega_M}{\omega_P}\right) \left(\frac{D_M}{D_P}\right)^3 \]
\[ \frac{H_M}{H_P} = \left(\frac{\omega_M}{\omega_P}\right)^2 \left(\frac{D_M}{D_P}\right)^2 \]
\[ \frac{P_M}{P_P} = \left(\frac{p_M}{p_P}\right) \left(\frac{\omega_M}{\omega_P}\right)^3 \left(\frac{D_M}{D_P}\right)^5 \]

Pump Specific Speed: \[ N_S = \omega Q^{1/2} / (gH)^{3/4} \]
Turbine Specific Speed: \[ N_S = \omega P^{1/2} / [\rho^{1/2} (gH)^{5/4}] \]
Moody Efficiency Relationship: \[ \eta_P = 1 - (1 - \eta_{IM}) \left(\frac{D_M}{D_P}\right)^{1/4} \left(\frac{H_M}{H_P}\right)^{1/10} \]
Power: \[ P = \rho g Q H \]

Pumps

Hydraulic Torque: \[ \tau = \rho Q (r_2 V_{2T} - r_1 V_{1T}) \]
Power: \[ P = 2\pi N \tau \]
Net Positive Suction Head: \[ \text{NPSH} = [(\rho_{atmosphere} - \rho_{vapour}) / \rho g] - \Delta z - h_L \]
Peripheral Velocity Factor: \[ \phi = V_{B2} / (2gh)^{1/2} \]
Critical Cavitation Parameter: \[ \sigma_C = \text{NPSH} / H \]

Steam Turbine

Force on Blades: \[ F = M(V_{S1}\cos\theta - V_{S2}\cos\delta) \]
Power to Blades: \[ P = M(V_{S1}\cos\theta - V_{S2}\cos\delta) V_B \]
Power to Blades: \[ P = M [(V_{S1}^2 - V_{S2}^2) + (V_{R2}^2 - V_{R1}^2)] / 2 \]