National Exams May 2012
07-Elec-A3, Signals and Communications
3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a Closed Book Exam but one aid sheet is allowed written on both sides. A Casio or Sharp approved calculator is permitted.

3. FIVE (5) questions constitute a complete paper. The first five questions as they appear in the answer book will be marked.

4. All questions are of equal value.

5. Clarity and organization of the answer are important.
1. (Total 20 marks) Consider a discrete-time linear time invariant system that is described by the following difference equation:

\[ \frac{1}{2} y[n - 2] - \frac{1}{4} y[n - 1] + y[n] = 2x[n - 1] + x[n], \]

where \( y[n] \) and \( x[n] \) are output and input sequences respectively.

(a) (10 marks) Find the impulse response of the system \( h[n] \).

(b) (10 marks) Draw the simulation block diagram in direct form II of the system.
2. (Total 20 marks) Figure 1. shows the exponential Fourier spectra of a periodic signal $x(t)$.

(a) (10 marks) By inspection of Fig. 1., find the exponential Fourier series representing $x(t)$.

(b) (5 marks) By inspection of Fig. 1., sketch the trigonometric Fourier spectra for $x(t)$.

(c) (5 marks) By inspection of the trigonometric Fourier spectra found in part (b), find the trigonometric Fourier series for $x(t)$.

Figure 1:
3. (Total 20 marks)

(a) (10 marks) Given the signal \( x(t) = e^{-2t}u(t) \), find and simplify the expression for the Fourier transform of the signal \( y(t) \), where

\[
y(t) = \int_{-\infty}^{t} [x(\lambda + 1) + x(\lambda - 2)]d\lambda.
\]

(b) (10 marks) Find and simplify the expression for the signal \( g(t) \) whose Fourier transform is given as:

\[
G(\omega) = \text{rect}(\frac{\omega}{4}) * [\delta(\omega + 10) + \delta(\omega - 10)]
\]
4. (Total 20 marks) The message \( x(t) = \text{sinc}^2(t) \) modulates the carrier \( \cos(10\pi t) \) to generate the modulated signal \( x_m(t) = x(t).\cos(2\pi f_c t) \).

(a) (6 marks) Sketch the spectrum of \( x(t) \), the carrier, and \( x_m(t) \).

(b) (7 marks) Sketch the spectrum of the LSB SSB signal.

(c) (7 marks) The signal in part (b) is synchronously demodulated. Sketch the spectrum of the demodulated signal.
5. (Total 20 marks) An FM signal is described by \( x(t) = A \cos[(2\pi \times 10^6 t + 50 \sin(2\pi \times 10^3 t))] \).

(a) (5 marks) Identify the carrier frequency.

(b) (7 marks) Identify the frequency of the modulating signal.

(c) (8 marks) Find the peak frequency deviation and modulation index.
6. (Total 20 marks) Consider a radio transmitter rated for $S_T \leq 3$ kW ($S_T$ is average transmitted power) and $A_{max}^2 \leq 8$ kW ($A_{max}^2$ is peak envelope power). Let the modulating signal be a tone with $A_m = 1$

(a) (5 marks) What is $S_x$, message power?

(b) (5 marks) If the modulation is DSB, what is the maximum possible power per sideband ($P_{sb}$)?

(c) (10 marks) Let the modulation signal be a square wave that switches periodically between $x(t) = +1$ and $x(t) = -1$. Sketch $x_c(t)$ when

i. the modulation is AM with $\mu = 0.5$ ($\mu$ is modulation index),

ii. the modulation is AM with $\mu = 1$, and

iii. the modulation is DSB.

Indicate the envelopes by dashed lines.