National Exams May 2012

07-Mec-A6-2  Advanced Strength of Materials

3 Hours Duration

NOTES:

1. If doubts exist as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Any non-communicating calculator is permitted. This is an open book exam.

3. Any five of the eight problems constitute a complete paper. If you choose to attempt more than five problems, only the first five problems as they appear in your answer book will be marked.

4. All problems are of equal value.
1. Under a given load, the 1.0 m by 1.5 m by 2.0 m parallelepiped shown below is deformed by movement of corner point A to a new location A' with coordinates (0.9975, 1.4988, 1.9991). If the displacement field is given by: \( u = c_{1}xyz \), \( v = c_{2}xyz \), \( w = c_{3}xyz \)

a. Determine \( \varepsilon_{x}, \varepsilon_{y}, \varepsilon_{z}, \gamma_{xy}, \gamma_{xz} \) and \( \gamma_{yz} \)

b. Evaluate the normal strain in the direction of line AB

c. Calculate the shear strain for perpendicular lines AB and AC.

2. The rods 1, 2, and 3 shown below are welded together, mounted between two rigid walls and subjected to the two forces shown at joints B and C. Rods 1 and 3 are of the same length, \( L_{1} = L_{3} = 2 \text{ m} \) and \( L_{2} = 2.5 \text{ m} \). Rods 1 and 3 are made from a material with \( E = 160 \text{ GPa} \). Rod 2 is made from a material with \( E = 70 \text{ GPa} \). The cross sections are given by: \( A_{1} = A_{3} = 10 \times 10^{3} \text{ mm}^{2} \) and \( A_{2} = 22 \times 10^{3} \text{ mm}^{2} \). Determine the displacements of joints B and C.
3. A state of plane stress is defined by the following stresses:

\[ \sigma_x = 140 \text{ MPa} \quad \sigma_y = 90 \text{ MPa} \quad \text{and} \quad \tau_{xy} = -40 \text{ MPa} \]

a. Assuming \(\sigma_{\text{yielding}} = 275 \text{ MPa}\), will this stress condition cause yielding according to the Von Mises Criterion?

b. Determine \(\sigma_x'\) and \(\tau_{xy}'\) on an element rotated 60 degrees clockwise from the x-axis.

4. An aluminum alloy bar of solid square cross-section (\(\sigma_{\text{yielding}} = 300 \text{ Mpa}\)) is subjected to a compressive axial force of magnitude \(P = 180 \text{ KN}\) acting at the centroid of the section and a torque \(T = 20 \text{ KN.m}\) as shown in the figure below. This member is to be designed in accordance with the maximum-shear-stress criterion of failure, with a safety factor of 2.

a. What is the minimum allowable cross-sectional dimension \(b\)?

b. What would your answer be if the load \(P\) was applied at the same point but parallel instead of normal to the cross section?

![Rigid Support](image)

5. A three element rosette is mounted on a thin metallic plate with a Young’s modulus of 70 GPa and a Poisson’s ratio of 0.25. The rosette provides the following readings along the 0, 45 and 90 degree directions respectively:

\[ \varepsilon_0 = 800 \mu \quad \varepsilon_{60} = 600 \mu \quad \varepsilon_{120} = 500 \mu \]

a. From these readings, calculate the strains \(\varepsilon_x', \varepsilon_y'\) and \(\gamma_{xy}'\) along the +45 degree direction.

b. Determine the principal strains \(\varepsilon_1\) and \(\varepsilon_2\) and the principal directions.

c. Using the generalized Hooke’s law, calculate \(\sigma_x, \sigma_y\) and \(\tau_{xy}\).
6. A 45000 N force is applied horizontally at joint B of the three-element, pin-joined truss shown below. Cross sections for all members are of $8 \text{ cm}^2$, $a = 90 \text{ cm}$, $b = 120 \text{ cm}$ and $E = 200 \text{ GPa}$. Determine the horizontal displacement $u$ and the vertical displacement $v$ at joint B.

7. Determine the magnitude and direction (up or down) of the force $P$ if the displacement at A is not to exceed 5 mm (down). Take $E = 210 \text{ GPa}$ and $I = 825 \times 10^6 \text{ mm}^4$.

8. A thick-walled cylinder with 0.20 m internal diameter and 0.30 m external diameter is fabricated of a material whose elastic limit is 350 MPa and Poisson's ratio $\nu = 0.33$. The cylinder is subjected to an internal pressure five times greater than the external pressure. Calculate the allowable internal pressure according to:

   a. the maximum shear stress theory, and

   b. the energy of distortion theory.