National Exams May 2013
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.

2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:

1. 20 marks
2. (a) 7 marks, (b) 7 marks, (c) 6 marks
3. (a) 6 marks, (b) 14 marks
4. 20 marks
5. 20 marks
6. (a) 7 marks, (b) 7 marks, (c) 6 marks
7. 20 marks
8. 20 marks
1. Solve the initial value problem

\[ y'' - 4y' + 8y = 5 \cos(2t), \quad y(0) = 0, \quad y'(0) = 0, \]

where \( \prime \) denotes differentiation with respect to \( t \).

2. Find the general solution of the following differential equations:

(a) \( y' + 2xy = 2xe^{-x^2}, \)
(b) \( y' + 2xy^2 = 0, \)
(c) \( y'' - 2y' + 3y = 0. \)

3. (a) Find the eigenvalues and the eigenvectors of the matrix \( A = \begin{pmatrix} 4 & 3 \\ -1 & 0 \end{pmatrix} \).
(b) Solve the system of differential equations

\[
\begin{align*}
\frac{dx}{dt} &= 4x + 3y, \\
\frac{dy}{dt} &= -x + t,
\end{align*}
\]

subject to the initial conditions \( x(0) = 2, \ y(0) = -1. \)

4. Let \( f(x, y) = 1 + x \ln(xy - 5) \). Find a formula for the plane tangent to the surface \( z = f(x, y) \) at the point \( (2, 3) \) and use the tangent plane to approximate \( f(2.1, 2.95) \).

5. Find the surface area of that portion of the surface \( z = 1 - \sqrt{x^2 + y^2} \) that lies in the first octant.

6. Consider the two lines defined as follows:

\[
\begin{align*}
x &= 3 - 2t, & y &= 3, & z &= 3 - t, \text{ (parameter } t) \\
x &= s, & y &= 1 - 2s, & z &= -s, \text{ (parameter } s) \end{align*}
\]

(a) Determine whether or not the two lines intersect, and if so, find the point of intersection.
(b) Find a third line orthogonal to both lines.
(c) Is there a plane containing both lines? If so, find an equation for that plane.

7. Evaluate the line integral \( \int_C \mathbf{v} \cdot d\mathbf{r} \) where \( C \) is the curve formed by the intersection of the cylinder \( x^2 + y^2 = 4 \) and the plane \( z + 2x - y = 3 \), travelled counterclockwise as viewed from the positive \( z \)-axis, and \( \mathbf{v} \) is the vector function \( \mathbf{v} = x \mathbf{i} + (x - y) \mathbf{j} + yz \mathbf{k} \).

8. Find the volume of the solid region inside the ellipsoid

\[ x^2 + y^2 + 4z^2 = 5 \]

and above the cone

\[ z = \sqrt{x^2 + y^2}. \]