NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a OPEN BOOK EXAM. Candidates my use only one of two approved calculators candidates are permitted.

3. Questions have equal value. The grade for each question is given. It is suggested that the candidate proportion time based on the allocated value.

4. All questions require an answer in analytical and/or essay format. Clarity and organization of the written answer and any figures or sketches are important.

5. The examination has an overall value of 80 Marks: each question will be marked out of 20 marks as per the marking scheme provided.

6. **ANSWER ONLY 4 of the 5 questions that are provided. Only the first 4 questions that appear in the answer book will be marked.**

7. Selected equations, graphs and tables are given at the end of the exam paper. These may (or may not) be of assistance for some questions. Indicate the question number corresponding to any graphs or tables used at the back of the exam question sheets.

8. Hand in the exam booklet and the question booklet at the end of the exam.
Marking Scheme

(only 4 will be marked)

1. 20 marks total
   (a) 10 marks
   (b) 5 marks
   (c) 5 marks
2. 20 marks total
3. 20 marks total
   20 marks total answer
4. 20 marks total
   (a) 10 marks
   (b) 10 marks
5. 20 marks total
   (a) 10 marks
   (b) 10 marks
**Value**

**20 Marks**

**Question #1**

Samples of a typical rock joint were tested in a square shear box of 160 mm x 160 mm dimensions, and the following data were collected:

<table>
<thead>
<tr>
<th>No.</th>
<th>$F_N$ (kN)</th>
<th>$F_{peak}$ (kN)</th>
<th>$F_{ult}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>1.3</td>
<td>2.5</td>
<td>0.8</td>
</tr>
<tr>
<td>#2</td>
<td>5.0</td>
<td>8.2</td>
<td>3.1</td>
</tr>
<tr>
<td>#3</td>
<td>10</td>
<td>13.6</td>
<td>5.6</td>
</tr>
<tr>
<td>#4</td>
<td>20</td>
<td>20.5</td>
<td>10.0</td>
</tr>
<tr>
<td>#5</td>
<td>30</td>
<td>25.1</td>
<td>13.6</td>
</tr>
<tr>
<td>#6</td>
<td>40</td>
<td>30.7</td>
<td>16.9</td>
</tr>
</tbody>
</table>

$F_N$ = Normal force  
$F_{peak}$ = Peak shear force  
$F_{ult}$ = Ultimate shear force

![Diagram](image)

**10 marks**  
**a.** Plot yield criteria for peak and ultimate strength on a Mohr diagram, noting that values in the Table are given in terms of force. Assume a Patton bilinear model (two straight lines), and fit the two parts of the Patton plot by hand, define the $c'$ and $\phi'$ of each part, and specify the approximate transition normal stress. Why does the ultimate yield criterion (in general) not show bilinearity?

**5 marks**  
**b.** Assuming that joints in all orientations exist, and that the principal total stresses are $\sigma_1 = 3$ MPa and $\sigma_3 = 1.2$ MPa, what pore pressure is required to just exceed the peak shearing criterion on the most critically oriented joint? (Use a Mohr-Coulomb construction). At this instant, what are the effective normal stresses and the shear stresses on the joint plane?

**5 marks**  
**c.** These tests were done on small specimens (0.0256 m$^2$). Discuss the issue of scale, as it relates to lab testing and field behaviour for rough joints.
Time and again, an emphasis has been placed on the issue of uncertainty in Rock Mechanics. It is a difficult issue to deal with, and because of this, past case histories, personal experience, and careful integration of the main factors in Geomechanical Design is required.

For the case of a horseshoe-shaped subway tunnel in horizontally bedded and jointed Ordovician limestones and non-swelling but plastic shales, 30 m under the city of Toronto, develop a pre-construction design strategy and a program during construction to cope with uncertainty. The following issues should be addressed, using diagrams, point-form, etc. The development of small flow charts may assist you in clarifying your answer, as design is largely a structured decision-making endeavour.

- Uncertainty in material parameters
- Probability of various "events" happening over the construction life
- Uncertainty in initial state in the ground and only scattered site investigation drillholes are available to you (one centreline drillhole per 100 metres length)
- Use of geophysical techniques to reduce uncertainty
- Adequacy of rock mechanics design in large openings
- Construction sequencing to reduce uncertainty
- Rock support strategies and their use
Question #3

The system of rock blocks shown in the sketch below is to be used in the verification procedure of a computer code for analysing progressive failure of rock slopes, and for this, a manually derived solution is required.

The system is in limiting equilibrium with block A tending to topple about the corner C, while block B is on the point of sliding downhill. The shear resistance on all surfaces is purely frictional with \( \varphi = 35^\circ \). Given that B is twice as heavy as A, determine the thickness ‘t’ of block B. Also show that there is no tendency for block A to slip at the corner C.

![Diagram of rock blocks](image)

Question #4

A servo-controlled compression test has been conducted on a weak soapstone such that the specimen length remained unchanged throughout: as the axial stress, \( \sigma_a \), was increased, so the confining pressure, \( p \), was increased so that no net axial strain resulted. A plot of axial stress (vertical axis) against confining pressure (horizontal axis) gave an initial straight line passing through the origin. At a critical confining pressure of \( p = 85 \) MPa (when \( \sigma_a = 39.1 \) MPa), the slope of the \( \sigma_a - p \) plot suddenly changed to \( 29^\circ \) and remained constant for the remainder of the test. This change in slope may be taken to represent the onset of yield. As such:

10 Marks

a. Determine an elastic constant from the slope of the initial portion of the \( \sigma_a - p \) curve.

10 Marks

b. Assuming that the Mohr-Coulomb criterion is applicable, determine \( \sigma_a, c \) and \( \varphi \) for the rock.
20 Marks  Question #5

If a rock mass contains more than one fracture set, one can apply the single plane of weakness theory to each set, and superimpose the results to find a lowest-bound envelope of strength. As such,

10 Marks  
a. Plot the 2-D variation in strength for a rock mass containing two orthogonal sets of fractures, A and B, the strengths of which are \( C_A = 100 \text{ kPa} \), \( \phi_A = 20^\circ \) and \( C_B = 0^\circ \), \( \phi_B = 35^\circ \) when the minor principal stress has the value 10 MPa. The intact rock strength is given by \( \sigma_1 = 75 + 5.29 \sigma_3 \).

10 Marks  
b. How would this strength variation change if the minor principal stress were reduced to zero?
Additional Reference Material

Equations

\[ RQD = 115 - 3.3 \ J_v, \]

Where, \( J_v \) is the sum of the number of joints per unit length for all joint (discontinuity) sets known as the volumetric joint count.

\[
Q = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF}
\]

where \( RQD \) is the Rock Quality Designation
\( J_n \) is the joint set number
\( J_r \) is the joint roughness number
\( J_a \) is the joint alteration number
\( J_w \) is the joint water reduction factor
\( SRF \) is the stress reduction factor

Resolved Normal Stress:

\[
\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{[(\sigma_x - \sigma_y)(\cos 2\theta)]}{2} + \tau_{xy}(\sin 2\theta)
\]

Resolved Shear Stress:

\[
\tau_\theta = \frac{[(\sigma_y - \sigma_x)(\sin 2\theta)]}{2} + \tau_{xy}(\cos 2\theta)
\]

Point Load Test

\[ l_{550} = \frac{L}{D^2} \]

Where, \( L \) = failure compressive loading force applied (kN);
\( D \) = specimen core diameter

\[ S_c = 24 \ l_{554} \text{ kPa} \]

Where, \( S_c \) = unconfined compressive strength (kPa)
\( (l_{554}) = \) index values for 5.4 cm diameter core specimens (kN/cm²)
Mohr Coulomb Failure Criterion

\[ \Psi = 45^\circ + \phi/2 \]

\[ S_T = C / \tan \phi \]

\[ (\sigma_1 + \sigma_3) / (\sigma_3 + S_T) = \tan^2 \Psi \]

\[ \sigma_1 = \sigma_3 \tan^2 \Psi + 2C \tan \Psi = \sigma_3 \tan^2 \Psi + S_c \]

Where, \( C \) = cohesion
\( \Psi \) = angle of failure plane in triaxial sample from horizontal
\( S_T \) = tensile strength
\( S_c \) = unconfined compressive strength

Mining

\[ \sigma_v = \text{load} / \gamma^2 \]

\[ \sigma_p = \text{load} / \chi^2 \]

\[ \frac{\sigma_p}{\sigma_v} = \frac{A_T}{A_P} \]

Where, \( A_p \) = Post mining area
\( A_T \) = Tributary Area

\[ \sigma_p = \frac{\sigma_v}{(1 - r)} \]

Where, \( r \) = extraction ratio = \( (A_T - A_p) / A_T \)

Kirsch Equations

\[ \sigma_{\tau\tau} = \sigma/2 \left\{ (1+k)(1-a^2/r^2) - (1-k)(1-4a^2/r^2 + 3a^4/r^4)\cos2\theta \right\} \]

\[ \sigma_{\theta\theta} = \sigma/2 \left\{ (1+k)(1+a^2/r^2) + (1-k)(1 + 3a^4/r^4)\cos2\theta \right\} \]

\[ \sigma_{r\theta} = \sigma/2 \left\{ (1-k)(1 + 2a^2/r^2 - 3a^4/r^4)\sin2\theta \right\} \]

\[ U_r = \{\mu \gamma/E\} \cdot \{(\sigma_1 + \sigma_3) + 2(\sigma_1 - \sigma_3)\cos2\theta \}

Where, \( \mu \) = Poisson's Ratio
Thick Wall Cylinder Stress formulae

\[(2P_o-P_i) = (P_i) \tan^2 \Psi + S_c\]

\[P_i = (2P_o - S_c) / (\tan^2 \Psi + 1)\]

\[\varepsilon_r = 1/E (\sigma_r - \mu \sigma_t) = U_r / r_i\]

\[U_r = \varepsilon_r r_i\]

\[U_r = \{\mu(2P_o r_i)\} / E\]

\[\sigma_t = 2(r_o^2 P_o) / (r_o^2 - r_i^2)\]

Where, \(P_o\) = pre-mining hydrostatic pressure at \(r = r_o\)

\(P_i\) = internal pressure applied against opening surface at \(r = r_i\)

\(\sigma_r\) = radially oriented post-mining stress components, uniform for all angular directions but varying by distance away from the excavation surface.

\(r_i\) = inside radius of circular opening in rock or liner\(\backslash\)

\(r_o\) = outside radius of installed liner or radial distance to boundary of rock media if the opening is unlined

\(\mu\) = Poisson's Ratio

\(U_r\) = inward radial displacement
### Table 1. Rock Mass Rating System (After Bieniawski 1989).

#### A. CLASSIFICATION PARAMETERS AND THEIR RATINGs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range of values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength of intact rock material</td>
<td>&gt;10 MPa</td>
</tr>
<tr>
<td>Unitary compressive strength</td>
<td>&gt;250 MPa</td>
</tr>
<tr>
<td></td>
<td>5 - 25 MPa</td>
</tr>
<tr>
<td>Rating</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Condition of discontinuities</th>
<th>Very rough surfaces</th>
<th>Slight rough surfaces</th>
<th>Slight weathered walls</th>
<th>Slight weathered walls or Soft gouge &lt; 5 mm thick or Separation &lt; 1-6 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>(See E)</td>
<td>No separation</td>
<td>Separation &lt; 1 mm</td>
<td>Highly weathered walls</td>
<td>Soft gouge &gt;5 mm thick or Separation &gt; 5 mm</td>
</tr>
<tr>
<td>Rating</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Groundwater</th>
<th>Inflow per 10 m tunnel length (litre)</th>
<th>&lt; 0.1</th>
<th>0.1 - 0.2</th>
<th>0.2 - 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>General conditions</td>
<td>Completely dry</td>
<td>Damp</td>
<td>Wet</td>
<td>Crumbling</td>
</tr>
<tr>
<td>Rating</td>
<td>15</td>
<td>10</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

#### B. RATING ADJUSTMENT FOR DISCONTINUITY ORIENTATION (See F)

<table>
<thead>
<tr>
<th>Strike and dip orientations</th>
<th>Favorable</th>
<th>Fair</th>
<th>Unfavorable</th>
<th>Very Unfavorable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunnels &amp; mines</td>
<td>0</td>
<td>-6</td>
<td>0</td>
<td>-12</td>
</tr>
<tr>
<td>Foundations</td>
<td>2</td>
<td>-7</td>
<td>-15</td>
<td>-26</td>
</tr>
<tr>
<td>Bases</td>
<td>-6</td>
<td>-25</td>
<td>-50</td>
<td></td>
</tr>
</tbody>
</table>

#### C. ROCK MASS CLASSES DETERMINED FROM TOTAL RATINGS

<table>
<thead>
<tr>
<th>Rating</th>
<th>Class number</th>
<th>Description</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 → 61</td>
<td>1</td>
<td>Very good rock</td>
<td>I</td>
</tr>
<tr>
<td>60 → 41</td>
<td>2</td>
<td>Good rock</td>
<td>II</td>
</tr>
<tr>
<td>40 → 21</td>
<td>3</td>
<td>Fair rock</td>
<td>III</td>
</tr>
<tr>
<td>20 → 10</td>
<td>4</td>
<td>Poor rock</td>
<td>IV</td>
</tr>
<tr>
<td>&lt; 20</td>
<td>5</td>
<td>Very poor rock</td>
<td>V</td>
</tr>
</tbody>
</table>

#### D. MEANING OF ROCK CLASSES

<table>
<thead>
<tr>
<th>Class number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Favorable</td>
</tr>
<tr>
<td>2</td>
<td>Fair</td>
</tr>
<tr>
<td>3</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>4</td>
<td>Very Unfavorable</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Averge drain-up time</th>
<th>20 yrs for 15 m span</th>
<th>1 year for 10 m span</th>
<th>1 week for 5 m span</th>
<th>10 min for 2.5 m span</th>
<th>30 min for 1 m span</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion of rock mass (kPa)</td>
<td>&gt;400</td>
<td>300 - 400</td>
<td>200 - 300</td>
<td>100 - 200</td>
<td>&lt; 100</td>
</tr>
<tr>
<td>Friction angle of rock mass (deg)</td>
<td>&gt;45</td>
<td>25 - 45</td>
<td>25 - 35</td>
<td>15 - 25</td>
<td>&lt; 15</td>
</tr>
</tbody>
</table>

#### E. GUIDELINES FOR CLASSIFICATION OF DISCONTINUITY conditions

<table>
<thead>
<tr>
<th>Discontinuity length (penetration)</th>
<th>&lt; 1 m</th>
<th>1 - 3 m</th>
<th>3 - 10 m</th>
<th>10 - 20 m</th>
<th>&gt; 20 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Separation (aperture)</td>
<td>None</td>
<td>&lt; 0.1 m</td>
<td>0.1 - 1.0 mm</td>
<td>1 - 5 mm</td>
<td>&gt; 5 mm</td>
</tr>
<tr>
<td>Rating</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Roughness</td>
<td>Very rough</td>
<td>Rough</td>
<td>Slight rough</td>
<td>Smooth</td>
<td>Stretched</td>
</tr>
<tr>
<td>Rating</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Jumbling (gouge)</td>
<td>None</td>
<td>Hard jumbling &lt; 5 mm</td>
<td>Hard jumbling &gt; 5 mm</td>
<td>Soft jumbling &lt; 5 mm</td>
<td>Soft jumbling &gt; 5 mm</td>
</tr>
<tr>
<td>Rating</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Weathering</td>
<td>Unweathered</td>
<td>Slightly weathered</td>
<td>Moderately weathered</td>
<td>Highly weathered</td>
<td>Decomposed</td>
</tr>
<tr>
<td>Rating</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

#### F. EFFECT OF DISCONTINUITY STRIKE AND DIP ORIENTATION IN TUNNELING**

<table>
<thead>
<tr>
<th>Strike perpendicular to tunnel axis</th>
<th>Strike parallel to tunnel axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive with dip - Dip 45 - 59°</td>
<td>Drive with dip - Dip 20 - 45°</td>
</tr>
<tr>
<td>Very Favorable</td>
<td>Favorable</td>
</tr>
<tr>
<td>Dip 45 - 60°</td>
<td>Dip 20 - 45°</td>
</tr>
<tr>
<td>Very Favorable</td>
<td>Favorable</td>
</tr>
<tr>
<td>DIP 0 - 10°</td>
<td>DIP 0 - 10°</td>
</tr>
<tr>
<td>Favor</td>
<td>Favor</td>
</tr>
</tbody>
</table>

*Some conditions are mutually exclusive. For example, if jumbling is present, the roughness of the surface will be overshadowed by the influence of the gouge. In such cases use A.4 directly.
Table 2. Guidelines for excavation and support of 10 m span rock tunnels in accordance with the RMR system (After Bieniawski 1989).

<table>
<thead>
<tr>
<th>Rock mass class</th>
<th>Excavation</th>
<th>Rock bolts (20 mm diameter, fully grouted)</th>
<th>Shotcrete</th>
<th>Steel sets</th>
</tr>
</thead>
</table>
| I - Very good rock  
*RMR: 81-100* | Full face, 3 m advance. | Generally no support required except spot bolting. | | |
| II - Good rock  
*RMR: 61-80* | Full face, 1-1.5 m advance. Complete support 20 m from face. | Locally, bolts in crown 3 m long, spaced 2.5 m with occasional wire mesh. | 50 mm in crown where required. | None. |
| III - Fair rock  
*RMR: 41-60* | Top heading and bench 1.5-3 m advance in top heading. Commence support after each blast. Complete support 10 m from face. | Systematic bolts 4 m long, spaced 1.5 - 2 m in crown and walls with wire mesh in crown. | 50-100 mm in crown and 30 mm in sides. | None. |
| IV - Poor rock  
*RMR: 21-40* | Top heading and bench 1.0-1.5 m advance in top heading. Install support concurrently with excavation, 10 m from face. | Systematic bolts 4-5 m long, spaced 1-1.5 m in crown and walls with wire mesh. | 100-150 mm in crown and 100 mm in sides. | Light to medium ribs spaced 1.5 m where required. |
| V - Very poor rock  
*RMR: < 20* | Multiple drifts 0.5-1.5 m advance in top heading. Install support concurrently with excavation. Shotcrete as soon as possible after blasting. | Systematic bolts 5-6 m long, spaced 1-1.5 m in crown and walls with wire mesh. Bolt invert. | 150-200 mm in crown, 150 mm in sides, and 50 mm on face. | Medium to heavy ribs spaced 0.75 m with steel lagging and forapoling if required. Close invert. |
Figures

Figure 1. RMR Rating System for the strength of intact rock material

Figure 2. The RMR Rating system: ratings for RQD
Figure 3. The RMR Rating system: ratings for Discontinuity Spacing
Figure 4. The RMR Rating system: Chart for correlation between RQD and Discontinuity Spacing.

Figure 5. Modified Lauffer diagram depicting boundaries of rock mass classes for TBM applications (after Lauffer 1988).
ROCK CLASSES

<table>
<thead>
<tr>
<th>G</th>
<th>F</th>
<th>E</th>
<th>D</th>
<th>C</th>
<th>B</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceptionally poor</td>
<td>Extremely poor</td>
<td>Very poor</td>
<td>Poor</td>
<td>Fair</td>
<td>Good</td>
<td>Very good</td>
</tr>
</tbody>
</table>

Bolt spacing in shotcreted areas

Span or height in m

ESR

Rock mass quality $Q = \frac{RQD}{Jn} \times \frac{Jr}{Ja} \times \frac{Jw}{SRF}$

REINFORCEMENT CATEGORIES:

1) Unsupported
2) Spot bolting
3) Systematic bolting
4) Systematic bolting, (and unreinforced shotcrete, 4 - 10 cm)
5) Fibre reinforced shotcrete and bolting, 5 - 9 cm
6) Fibre reinforced shotcrete and bolting, 9 - 12 cm
7) Fibre reinforced shotcrete and bolting, 12 - 16 cm
8) Fibre reinforced shotcrete, > 15 cm.
   reinforced rib of shotcrete and bolting
9) Cast concrete lining

Figure 6. Estimated support categories based on the tunnelling quality index Q (After Grimstad and Barton, 1993, reproduced from Palmstrom and Broch, 2006).