Notes:
1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.

2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:
1. (a) 10 marks, (b) 10 marks
2. 20 marks
3. 20 marks
4. (a) 8 marks, (b) 12 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks
1. For each of the following differential equations, find the general solution, \( y(x) \).

   (a) \( y'' + 9y = \sec 3x \)

   (b) \( y'' - y' - 6y = 3x^2 + e^{-2x} \)

   Note that ‘ denotes differentiation with respect to \( x \).

2. Find the maximum and minimum values of \( f(x,y,z) = x + y - z \) over the sphere \( x^2 + y^2 + z^2 = 1 \).

3. Find the line tangent to the intersection of the surfaces

   \[ 3x^2 + 2y^2 - 2z = 1 \]

   and

   \[ x^2 + y^2 + z^2 - 4y - 2z + 2 = 0 \]

   at the point \((1, 1, 2)\).

4. Let \( A = \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix} \).

   (a) Find the eigenvalues and eigenvectors of \( A \).

   (b) Solve the initial value problem

   \[
   \begin{align*}
   x' &= 3x + y, & x(0) &= 1, \\
   y' &= -2x + y, & y(0) &= 0.
   \end{align*}
   \]

5. Evaluate the surface integral \( \iint_S \mathbf{F} \cdot dS \) where \( \mathbf{F}(x,y,z) = yz\mathbf{i} - 2xy\mathbf{j} + 3z\mathbf{k} \) and \( S \) is the surface of the region bounded above by the paraboloid \( z = 4 - x^2 - y^2 \) and below by the plane \( z = 0 \).

6. Find the volume of the region bounded by the paraboloid \( z = \frac{x^2}{4} + \frac{1}{2}(x^2 + y^2) \) and the plane \( z = 4 \) that lies outside the cone \( z^2 - 4x^2 - 4y^2 = 0 \).

7. Let \( C \) be the curve formed by the intersection of the cylinder \( x^2 + y^2 = 9 \) and the plane \( z = 1 + y - 2x \), travelled clockwise as viewed from the positive \( z \)-axis, and let \( \mathbf{v} \) be the vector function \( \mathbf{v} = 4zi - 2yj + 2yk \). Evaluate the line integral \( \int_C \mathbf{v} \cdot dr \).

8. Compute the response of the damped mass-spring system modelled by

   \[
   y'' + 3y' + 2y = r(t), \quad y(0) = 0, \quad y'(0) = 0,
   \]

   where \( r \) is the square wave

   \[
   r(t) = \begin{cases} 
   1, & 1 \leq t < 2, \\
   0, & \text{otherwise},
   \end{cases}
   \]

   and ‘ denotes differentiation with respect to time.