NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is an OPEN BOOK EXAM. Any non-communicating calculator is permitted.

3. FIVE (5) questions constitute a complete exam paper. The first five questions as they appear in the answer book will be marked.

4. Each question is of equal value.

5. Most questions require an answer in essay format. Clarity and organization of the answer are important.
Two tanks are connected in series in a noninteracting fashion as shown in the figure.

Assume: \( \rho = 1 \quad A = 1 \) (A-cross-section of each tank)

\[
q_2 = \frac{1}{R} \sqrt{\frac{\Delta p}{\rho g}} \quad \text{and} \quad q_1 \quad \text{is determined by a pump.}
\]

The initial value of the inlet flowrate is \( q_{in} = 10 \) and remains constant. The initial level in tank 1 is \( h_1(t=0) = 10 \). \( q_1 \) is the manipulated variable. All q's are volumetric flow rates. \( R = 2 \).

(10%) (a) Show the differential equations that describe the behaviour of \( h_1(t) \) and \( h_2(t) \).

(10%) (b) Compute transfer functions between \( h_1 \) to \( q_{in} \) and \( h_2 \) to \( q_{in} \).
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PROBLEM #2 (20% total)

For the draining tank shown in the figure:

Compute the change in level $\delta h(t)$ as a function of time for the following two cases:

(10%) (a) a step of one unit in inlet flow $F_0$

(10%) (b) a unit impulse (Dirac function) in inlet flow $F_0$

The cross-section area of the tank is 1 m$^2$. The length of the inlet pipe is 1.0m and the cross sectional area of the pipe is 0.01m$^2$. Initial level = 1 m.

The flow out is given by $F_1 = R_1 \cdot h$, where the hydraulic resistance $R_1 = \frac{1 m^2}{\text{min}}$. 
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**PROBLEM #3** (20% total)

A process is described by the following transfer function:

\[ G_p = \frac{10(1-s)e^{-10s}}{100s + 1} \]

(10%) (a) Design an IMC (Internal Model Controller) for this process. Show your design with a block diagram.

(10%) (b) Assuming a perfect model of the process, compute the closed loop response for a unit step in set point if the desired closed loop time constant is equal to 10.

**PROBLEM #4** (20% total)

A process given by:

\[ G_p = \frac{100}{s - 10} \]

is controlled by a proportional controller with gain \( K_c \).

(10%) (a) Using the Nyquist theorem test the closed loop stability for \( K_c = 1 \) and \( K_c = 0.01 \).

(10%) (b) Using the Nyquist criterion, compute the limiting value of \( K_c \) for which the system is stable.
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PROBLEM # 5 (20%)

A thermocouple is immersed in a well stirred bath of liquid. The geometry and properties of the thermocouple's material are as follows:

mass = 0.25 g
heat capacity = 1 cal/g°C

Heat transfer coefficient between the thermocouple and the liquid = 60 cal/cm² h°C surface area of the thermocouple = 1 cm².

(10%) 1. Find the transfer function that relates the temperature of the thermocouple to the temperature in the liquid. Assume that there are no gradients in the thermocouple bead, no conduction through the thermocouple wires and the conversion from millivolt to degrees occurs by a very fast reading device.

(10%) 2. If the temperature in the liquid changes according to the following diagram:

\[
\begin{align*}
T_{\text{liquid}} & \quad \text{time} \\
300°C & \quad 300 s \quad 600 s
\end{align*}
\]

Calculate the temperature registered by the thermocouple as a function of time.
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PROBLEM #6 (20% total)

For the equation

\[ \frac{d^2 y}{dt^2} + k \frac{dy}{dt} + 10y = 2x \]

(10%) (a) Find the transfer function between the input x to the output y and put it in the standard gain-time constant form.

(5%) (b) Discuss for which values of k is the open loop response for a unit step in x (i) stable, (ii) underdamped, and (iii) overdamped.

(5%) (c) If the response is underdamped, compute expressions as a function of k for the time constant and the damping coefficient according to the standard form definitions.

PROBLEM #7 (20% total)

The dynamic response of the reactant concentration in a CSTR reactor, \( C_A \), to a change in inlet concentration, \( C_{A_0} \), has to be evaluated.

The reactor is operated with constant volume \( V \) and isothermal conditions. The density \( \rho \) is constant.

The reaction rate is:

\[ r_A = k_i C_A^2 \]

The mass flowrate is F.

(10%) (a) Derive a mathematical model to describe \( C_A(t) \) and compute steady state conditions for concentration.

(10%) (b) Compute a transfer function \( \delta C_A / \delta C_{A_0} \) (where \( \delta \) indicates deviation variables) when the system is operated around the steady state computed in (a).
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PROBLEM #8 (20% total)

A process is described by the following transfer function:

\[ G_p = \frac{e^{-2s}}{(s+1)} \]

The process is controlled by a proportional controller with gain \( K_c \).

(10%) (a) Plot qualitatively the gain and phase diagrams for \( K_c G_p(s) \). Indicate “corner” frequencies, asymptotic values of gain and phase angles and slope values.

(10%) (b) Compute analytically the gain margin for \( K_c = 1 \).