NATIONAL EXAMINATIONS

May 2014

07-MEC-A6-1 FLUID MACHINERY

Three hours duration

Notes to Candidates

1. This is a Closed Book examination.

2. Exam consists of two Sections Section A is Calculative (5 questions) and Section B is Descriptive (3 questions).

3. Do four (4) questions (including all parts of each question) from Section A (Calculative) and two (2) questions from Section B (Descriptive).

4. Note that Question 2 is on two pages.

5. Six questions constitute a complete paper. (Total 60 marks).

6. All questions are of equal value. (Each 10 marks).

7. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

8. If any initial parts of a multi-part question cannot be solved the remaining parts may be worked by making appropriate assumptions for the first parts from the technical data given.

9. Candidates may use one of the approved Casio or Sharp calculators.

10. Reference data for particular questions are given in the Attachments on pages 11 to 17. All pages on which answers have been written are to be returned with the answer booklet. Candidate's names must be on these sheets.

11. Reference formulae and constants are given on pages 18 to 22.

12. Drawing Instruments (scale ruler, protractor and sharp pencil) are required for vector diagrams.
SECTION A  CALCULATIVE QUESTIONS

QUESTION 1  PUMP POWER AND EFFICIENCY

PART I  PUMP POWER REQUIREMENT

The diameters of the suction and discharge pipes of a pump are 150 mm and 100 mm, respectively. The discharge pressure is read by a gauge at a point 1.5 m above the centre line of the pump, and the suction pressure is read by a gauge 0.5 m below the centre line. The pressure gauge reads a pressure of 150 kPa and the suction gauge reads a vacuum of 30 kPa (negative gauge pressure) when gasoline having a specific gravity of 0.75 is pumped at the rate of 0.035 m³/s. Calculate the electrical power required to pump the fluid if the pump efficiency is 75%.

( 5 marks )

PART II  HOMOLOGOUS PUMP SCALING

A one tenth (1/10) scale model pump impeller with a diameter of 188 mm is tested in a test facility using water under the following conditions:

<table>
<thead>
<tr>
<th>Pump Speed</th>
<th>3600 rev/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>39.6 m</td>
</tr>
<tr>
<td>Flow</td>
<td>0.085 m³/s</td>
</tr>
</tbody>
</table>

Under these conditions the model impeller was found to have an efficiency of 84%.

A homologous prototype (with an impeller 10 times the diameter of the model and geometrically identical to it) is to be installed and operated under a head of 110 m while pumping water.

Determine the following:

(a) Speed at which the prototype pump would need to operate to ensure homologous conditions.

(b) Flow rate of water through the pump under homologous conditions.

(c) Power required to operate the pump assuming ideal conditions.

(d) Efficiency anticipated under the above conditions.

( 5 marks )

[ 10 marks ]
QUESTION 2 HYDRO TURBINES

PART I PELTON WHEEL

Refer to the Examination Paper Attachments Page 11 Bridge River Plant.

Note: Convert the given data to SI units using the conversions below and solve in SI units.

\[
\begin{align*}
1 \text{ HP} &= 746 \text{ watts} \\
1 \text{ inch} &= 25.4 \text{ mm}
\end{align*}
\]

Data:  
| Gross head  | 1 226 ft |
| Net head    | 1 118 ft |
| Power output| 62 000 HP |
| Rotational speed | 300 rev/min |
| Pitch diameter | 95 in |

Determine the following:

(a) Ratio of actual blade velocity to anticipated jet velocity.

(b) Deviation as a percentage of the ratio calculated in (a) above from the ideal ratio and give a possible reason for this deviation.

(c) Volume flow rate required to give the specified output.

(5 marks)

This question is continued on the next page
QUESTION 2  (Continued)

PART II  TURBINE SETTING

Refer to the Examination Paper Attachments Page 12 Critical Cavitation Parameter.

Vandekloof Hydro Power Station has the following technical parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical generator design output</td>
<td>120 MW</td>
</tr>
<tr>
<td>Electrical generator voltage</td>
<td>11 kV</td>
</tr>
<tr>
<td>Speed of turbine-generator</td>
<td>125 rev/min</td>
</tr>
<tr>
<td>Type of hydro turbine</td>
<td>Francis</td>
</tr>
<tr>
<td>Design head on turbine</td>
<td>65 m</td>
</tr>
<tr>
<td>Maximum water consumption (at lower head)</td>
<td>217 m³/s</td>
</tr>
<tr>
<td>Inlet diameter to spiral casing</td>
<td>7 m</td>
</tr>
<tr>
<td>Turbine runner diameter</td>
<td>5462 mm</td>
</tr>
<tr>
<td>Turbine runner material</td>
<td>Stainless cast steel</td>
</tr>
</tbody>
</table>

(a) Calculate the specific speed of the turbine.

(b) From the graph determine the Thoma cavitation parameter $\sigma$.

(c) Calculate the setting (maximum elevation) of the turbine runner relative to the tailrace water level based on the critical cavitation parameter (Thoma coefficient).

( 5 marks )

[ 10 marks ]
QUESTION 3 STEAM TURBINE BLADES

Refer to the Examination Paper Attachments Page 13 Steam Turbine Velocity Diagram

The attached diagram clarifies the nomenclature used in the question below. Use this same nomenclature in your answer.

One stage of a steam turbine operating on the impulse principle has the following blade characteristics:

- Moving blade velocity $V_b = 100\text{ m/s}$
- Inlet steam velocity $V_{s1} = 300\text{ m/s}$
- Nozzle exit angle $\theta = 25^\circ$
- Steam mass flow rate $M = 24\text{ kg/s}$

The moving blades are assumed to be symmetrical and frictionless, that is:

- Blade outlet angle $\gamma = \text{Blade inlet angle } \phi$
- Relative velocity $V_{r2} = \text{Relative velocity } V_{r1}$

Draw a velocity (vector) diagram (see note below) to a scale of 1 cm = 20 m/s to show the absolute and relative velocities within the turbine blades. By measuring from this diagram determine the following:

- (a) Absolute exhaust steam velocity.
- (b) Impulse force on the moving blades.
- (c) Energy transferred to the moving blades in kJ/kg.
- (d) Inlet and exhaust kinetic energies in kJ/kg.
- (e) Blade efficiency.
- (f) Power developed by the turbine stage.

Note: While calculation of velocities by trigonometric ratios with reference to a sketch is acceptable it is longer and more time consuming.

[10 marks]
QUESTION 4  COMPRESSOR FIRST STAGE

Refer to the Examination Paper Attachments Page 14 Acacia and Port Rex Power Stations and Page 15 Compressor Velocity Diagram.

Each power station has three units. Each unit has an output of 60 MW and is powered by twin back to back gas turbines driving a common electrical generator. The diagram and specifications on Page 14 are for one gas turbine only.

Consider the first stage of the compressor (N1 rotor) which has the following approximate parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor hub diameter at inlet</td>
<td>$D_1 = 480$ mm</td>
</tr>
<tr>
<td>Blade tip diameter at inlet</td>
<td>$D_2 = 1120$ mm</td>
</tr>
<tr>
<td>Inlet guide vane outlet angle</td>
<td>$\alpha_1 = 30^\circ$</td>
</tr>
<tr>
<td>First stage moving blade outlet angle</td>
<td>$\beta_2 = 40^\circ$</td>
</tr>
</tbody>
</table>

Assume that the plant is operating under the following conditions:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational speed</td>
<td>$N = 6800$ rev/min</td>
</tr>
<tr>
<td>Air flow at inlet to compressor</td>
<td>$M = 136$ kg/s</td>
</tr>
<tr>
<td>Air temperature at inlet to compressor</td>
<td>$T_1 = 15^\circ C \ (288^\circ K)$</td>
</tr>
</tbody>
</table>

Assume that ideal conditions prevail (no friction losses).

(a) Calculate the blade velocity $U$ and air inlet axial velocity $C_{x1}$.

(b) Draw to scale (see note below) the velocity diagrams at the first stage moving blade inlet and outlet and measure the absolute velocities $C_1$ and $C_2$ and relative velocities $W_1$ and $W_2$.

(c) Determine the work done in kJ/kg and power input to the first stage in kW.

(d) Determine the enthalpy rise and hence temperature rise in the first stage.

(e) Determine the pressure ratio of the first stage assuming isentropic conditions.

Note: The scale drawing should be to a large enough scale for accurate measurements (a scale of 10 m/s = 4 mm is suggested). While calculation of velocities by trigonometric ratios is acceptable it is longer and more time consuming.

[ 10 marks ]
QUESTION 5  BOILER DRAUGHT FANS

Two induced draught (ID) fans are employed in parallel at the exhaust of a large coal fired boiler to extract the combustion gases. Each fan has the following head versus flow characteristic where \( H \) is in kPa and \( Q \) is in m\(^3\)/s:

\[
H = K_1 - K_2 Q - K_3 Q^2
\]

The complete exhaust system has the following head versus flow characteristic where \( h \) is in kPa and \( Q \) is in m\(^3\)/s

\[
h = K_4 Q^2
\]

The constants and full load speed of the fans are as follows where rotational speed \( N \) is in rev/min:

\[
\begin{align*}
K_1 &= 4.5 \times 10^{-6} N^2 \\
K_2 &= 0.0 \\
K_3 &= 16.0 \times 10^{-6} \\
K_4 &= 5.5 \times 10^{-6} \\
N &= 1155 \text{ rev/min}
\end{align*}
\]

Sketch the following:

(a) Head versus flow characteristics with one fan in operation and with both fans in operation. In each case identify the operating point of the system.

Calculate the following:

(b) Volume flow rate of exhaust gas with only one fan in operation.

(c) Volume flow rate of exhaust gas with both fans in operation (to give maximum load on the boiler).

(d) Load possible on the boiler with one fan in operation (as a percentage of maximum load as calculated in (c) above).

(e) Speed requirement of both fans to give together the same load as is possible with only one fan in operation (as in (b) above).

[ 10 marks ]
SECTION B  DESCRIPTIVE QUESTIONS

Note that each question requires a detailed answer with complete explanations, with sketches or diagrams if appropriate, to support the explanation. A ten mark descriptive question requires an answer of approximately two full pages.

While each part of each question may specify several aspects, more emphasis may be put on one or more aspects and less on others provided an overall comprehensive answer is given.

QUESTION 6

Refer to the Examination Paper Attachments Page 16  Hydraulic Efficiencies of Turbines.

The diagrams show the variation in efficiency with load of the following hydro turbines:

- Kaplan Turbine
- Francis Turbine
- Pelton Turbine

Explain why the efficiencies change with load as they do and why the efficiencies are different for different turbines. Explain also the choice of machines for different applications. In particular explain the following;

(a) Why the efficiency of the Francis turbine is the best and that of the Pelton turbine the worst at high load conditions.

(b) Why the Kaplan turbine efficiency remains high to quite low loads.

(c) Why the Francis turbine efficiency drops quite soon as load is decreased.

(d) Why the Pelton turbine has a high efficiency even at very low loads.

(e) What determines the choice of turbine type for a particular application and what is the reason for this.

[ 10 marks ]
QUESTION 7  FAN CONTROL

Refer to the Examination Paper Attachments Page 17 Fan Control Methods.

The diagrams show (as dotted lines) the system and fan characteristics and operating point for normal design conditions.

The volume flow rate through the system can be controlled (reduced) by three methods:

- Dampers in the ducting which may be progressively closed.
- Vanes at the fan inlet which can create increasing pre-whirl.
- Speed of driving motor which can progressively reduce fan speed.

(a) For each of these three methods show on the diagrams how the system or fan characteristics change to give a new operating point.

   (i) Control by duct dampers.

   (ii) Control by inlet vanes.

   (iii) Control by fan speed.

   In each case show the new operating point.

*Return this page with the examination answer booklet.*

( 5 marks )

(b) Explain, with reference to the diagrams, why and how the flow is reduced in each case.

   (i) Effect of duct dampers.

   (ii) Effect of inlet vanes.

   (iii) Effect of fan speed.

*Hint: Sketches of velocity diagrams may be useful in the explanation for (ii).*

( 5 marks )

[ 10 marks ]
QUESTION 8  NUMBER OF STAGES

Refer to the Examination Paper Attachments Page 14 Acacia and Port Rex Power Stations.

The cross section of the gas turbines shows the number of stages in the compressor and the number of stages in the turbine. Explain why the compressor has many more stages than the turbine. Give the reasons for selecting the appropriate number of stages for both the compressor and the turbine. In particular explain what requires the compressor to have a certain minimum number of stages and why the turbine must have multiple stages.

[ 10 marks ]
EXAMINATION PAPER ATTACHMENTS

QUESTION 2 PART 1  BRIDGE RIVER PLANT

484  15 Impulse Turbines

Figure 15.2  Vertical-shaft impulse turbine with six nozzles at Bridge River plant in British Columbia. Gross head = 1,226 ft, net head = 1,118 ft, 62,000 hp, n = 300 rpm, pitch diameter = 95 in.
EXAMINATION PAPER ATTACHMENTS

QUESTION 2  CRITICAL CAVITATION PARAMETER

Variation of critical cavitation parameter with non-dimensional specific speed in SI units for Francis and Kaplan turbines
QUESTION 3  STEAM TURBINE VELOCITY DIAGRAM

Nomenclature for velocity vectors and angles

\[ V_{S1} \] Absolute steam velocity entering moving blades
\[ V_{R1} \] Relative steam velocity entering moving blades
\[ V_B \] Moving blade velocity
\[ V_{R2} \] Relative steam velocity leaving moving blades
\[ V_{S2} \] Absolute steam velocity leaving moving blades
EXAMINATION PAPER ATTACHMENTS

QUESTION 4 & QUESTION 8  ACACIA AND PORT REX POWER STATIONS

Technical Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Peak Load</th>
<th>Base Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Output (kW)</td>
<td>60 860</td>
<td>57 100</td>
</tr>
<tr>
<td>Heat Rate (kJ/kWh)</td>
<td>11 791</td>
<td>11 887</td>
</tr>
<tr>
<td>Speed - N1 Rotor (rev/min)</td>
<td>6 805</td>
<td>6 640</td>
</tr>
<tr>
<td>Speed - N2 Rotor (rev/min)</td>
<td>8 395</td>
<td>8 320</td>
</tr>
<tr>
<td>Speed - Power Turbine (rev/min)</td>
<td>3 000</td>
<td>3 000</td>
</tr>
<tr>
<td>Temperature - Gas Generator Turbine Inlet (°C)</td>
<td>1 077</td>
<td>1 043</td>
</tr>
<tr>
<td>Temperature - Power Turbine Inlet (°C)</td>
<td>682</td>
<td>657</td>
</tr>
<tr>
<td>Temperature - Power Turbine Exhaust (°C)</td>
<td>483</td>
<td>467</td>
</tr>
<tr>
<td>Exhaust Gas Flow Rate (kg/s)</td>
<td>278</td>
<td>272</td>
</tr>
<tr>
<td>Gas Generator Pressure Ratio</td>
<td>14.1</td>
<td>13.6</td>
</tr>
</tbody>
</table>

N1 Low Speed Compressor and Turbine
N2 High Speed Compressor and Turbine

Inlet Air Conditions  15°C
QUESTION 4  COMPRESSOR VELOCITY DIAGRAM

Rotor Blade Row

Stator Blade Row

\( \text{U} \)  Blade velocity
\( \text{C}_1 \)  Rotor blade absolute inlet velocity
\( \text{W}_1 \)  Rotor blade relative inlet velocity
\( \text{C}_2 \)  Rotor blade absolute outlet velocity
\( \text{W}_2 \)  Rotor blade relative outlet velocity
\( \text{C}_3 \)  Stator blade absolute outlet velocity
Fig. 9.14. Variation of hydraulic efficiency for various types of turbine over a range of loading, at constant speed and constant head.
QUESTION 7  FAN CONTROL METHODS

(i) Control by duct dampers

(ii) Control by inlet vanes

(iii) Control by fan speed
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Flow area, Surface area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure</td>
<td>J/kg$^\circ$C</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Specific heat at constant volume</td>
<td>J/kg$^\circ$C</td>
</tr>
<tr>
<td>b</td>
<td>Width</td>
<td>m</td>
</tr>
<tr>
<td>C</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>D</td>
<td>Diameter</td>
<td>m</td>
</tr>
<tr>
<td>E</td>
<td>Energy</td>
<td>J</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
<td>N</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>h</td>
<td>Specific enthalpy</td>
<td>J/kg</td>
</tr>
<tr>
<td>h_s</td>
<td>System head</td>
<td>m</td>
</tr>
<tr>
<td>h_L</td>
<td>Head loss</td>
<td>m</td>
</tr>
<tr>
<td>H</td>
<td>Pump or turbine head</td>
<td>m</td>
</tr>
<tr>
<td>k</td>
<td>Ratio of specific heats</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Length</td>
<td>m</td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
<td>kg</td>
</tr>
<tr>
<td>M</td>
<td>Mass flow rate</td>
<td>kg/s</td>
</tr>
<tr>
<td>N</td>
<td>Rotational speed</td>
<td>rev/s</td>
</tr>
<tr>
<td>N_s</td>
<td>Specific Speed</td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>Pressure</td>
<td>Pa (N/m$^2$)</td>
</tr>
<tr>
<td>P</td>
<td>Power</td>
<td>W (J/s)</td>
</tr>
<tr>
<td>q</td>
<td>Heat transferred</td>
<td>J/kg</td>
</tr>
<tr>
<td>Q</td>
<td>Heat</td>
<td>J</td>
</tr>
<tr>
<td>Q</td>
<td>Flow rate</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>r</td>
<td>Radius</td>
<td>m</td>
</tr>
<tr>
<td>R</td>
<td>Specific gas constant</td>
<td>J/kg K</td>
</tr>
<tr>
<td>s</td>
<td>Entropy</td>
<td>J/kg K</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>u</td>
<td>Specific internal energy</td>
<td>J/kg</td>
</tr>
<tr>
<td>U</td>
<td>Internal Energy</td>
<td>J</td>
</tr>
<tr>
<td>U</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>v</td>
<td>Specific volume</td>
<td>m$^3$/kg</td>
</tr>
<tr>
<td>V</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>W</td>
<td>Specific work</td>
<td>J/kg</td>
</tr>
<tr>
<td>W</td>
<td>Work</td>
<td>J</td>
</tr>
<tr>
<td>x</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>z</td>
<td>Length</td>
<td>m</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Pump blade angle</td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Compressor blade angle</td>
<td></td>
</tr>
<tr>
<td>(\beta)</td>
<td>Pump blade angle</td>
<td></td>
</tr>
<tr>
<td>(\beta)</td>
<td>Compressor blade angle</td>
<td></td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Turbine blade angle</td>
<td></td>
</tr>
<tr>
<td>(\phi)</td>
<td>Turbine blade angle</td>
<td></td>
</tr>
<tr>
<td>(\delta)</td>
<td>Turbine blade angle</td>
<td></td>
</tr>
<tr>
<td>(\eta)</td>
<td>Efficiency</td>
<td>Ns/m²</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Nozzle angle</td>
<td>m²/s</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Dynamic viscosity</td>
<td>kg/m³</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Kinematic viscosity</td>
<td>N</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density</td>
<td>Nm</td>
</tr>
<tr>
<td>(\sigma_c)</td>
<td>Critical cavitation parameter</td>
<td>rad/s</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Thrust</td>
<td>J/s</td>
</tr>
<tr>
<td>(\sigma_c)</td>
<td>Torque</td>
<td></td>
</tr>
<tr>
<td>(\phi)</td>
<td>Peripheral velocity factor</td>
<td></td>
</tr>
<tr>
<td>(\omega)</td>
<td>Rotational speed</td>
<td></td>
</tr>
<tr>
<td>(\Omega)</td>
<td>Heat transfer rate</td>
<td></td>
</tr>
</tbody>
</table>

**GENERAL CONSTANTS**

*Use unless otherwise specified*

- Acceleration due to gravity: \(g = 9.81\, \text{m/s}^2\)
- Atmospheric pressure: \(p_{\text{atm}} = 100\, \text{kPa}\)
- Water vapour pressure: \(p_{\text{vapour}} = 2.34\, \text{kPa}\) (at 20°C)
- Density of water: \(\rho_{\text{water}} = 1000\, \text{kg/m}^3\)
- Density of air: \(\rho_{\text{air}} = 1.21\, \text{kg/m}^3\) (at 15°C)
- Density of air: \(\rho_{\text{air}} = 1.19\, \text{kg/m}^3\) (at 20°C)
- Specific heat of air: \(c_p = 1.005\, \text{kJ/kg°C}\)
- Specific heat of air: \(c_v = 0.718\, \text{kJ/kg°C}\)
- Specific heat of water: \(c_p = 4.19\, \text{kJ/kg°C}\)
GENERAL REFERENCE EQUATIONS

Basic Thermodynamics

First Law: \[ dE = \delta Q - \delta W \]
Enthalpy: \[ h = u + pv \]
Continuity: \[ pVA = \text{constant} \]
Potential Energy: \[ E_{PE} = mgz \]
Kinetic Energy: \[ E_{KE} = \frac{V^2}{2} \]
Internal Energy: \[ E_{IN} = U \]
Flow Work: \[ w = \Delta(pv) \]
Energy Equation: \[ zg + \frac{V^2}{2} + u + pv + \Delta w + \Delta q = \text{constant} \]

Ideal Gas Relationships

Gas Law: \[ pv = RT \]
Specific Heat at Constant Pressure: \[ c_p = \frac{\Delta h}{\Delta T} \]
Specific Heat at Constant Volume: \[ c_v = \frac{\Delta u}{\Delta T} \]
Specific Gas Constant \[ R = c_p - c_v \]
Ratio of Specific Heats \[ k = \frac{c_p}{c_v} \]
Isentropic Relations: \[ \frac{p_1}{p_2} = (\frac{V_2}{V_1})^k = (\frac{T_2}{T_1})^{k-1} \]

FLUID MACHINERY REFERENCE EQUATIONS

Fluid Mechanics

Pressure \[ p = \rho gh \]
Continuity Equation: \[ p_1V_1A_1 = p_2V_2A_2 = M \]
Bernoulli’s Equation: \[ p_1/\rho g + z_1 + \frac{V_1^2}{2g} = p_2/\rho g + z_2 + \frac{V_2^2}{2g} \]
Momentum Equation: \[ F = p_1A_1 - p_2A_2 - \rho VA(V_2 - V_1) \] (one dimensional)

Steam Turbines

Nozzle Equation: \[ h_1 - h_2 = \frac{(V_2^2 - V_1^2)}{2} \]
Work: \[ w = \frac{[(V_{1,\text{absolute}}^2 - V_{2,\text{absolute}}^2) + (V_{2,\text{relative}}^2 - V_{1,\text{relative}}^2)]}{2} \]
Work: \[ w = (V_{S1}\cos\theta - V_{S2}\cos\delta) V_{\text{blade}} \]
Power: \[ P = wM \]
Gas Turbines

State Equation: \( pv = RT \)
Isentropic Equation: \((T_2/T_1) = (p_2/p_1)^{(k-1)/k}\)
Enthalpy Change: \( h_1 - h_2 = c_p(T_1 - T_2) \) (ideal gas)
Nozzle Equation: \( h_1 - h_2 = (V_2^2 - V_1^2) / 2 \)
Work: \( w = (C_1 \sin \alpha_1 + C_2 \sin \alpha_2) U \)
Power: \( P = wM \)

Compressors

Work \( W = U(C_v^2 - C_y^1) \)
Rotor Enthalpy Change \( h_1 + \frac{1}{2}W_1^2 = h_2 + \frac{1}{2}W_2^2 \)
Stator Enthalpy Change \( h_2 + \frac{1}{2}C_2^2 = h_3 + \frac{1}{2}C_3^2 \)
Isentropic Equation: \((T_3/T_1) = (p_3/p_1)^{(k-1)/k}\)

Jet Propulsion

Thrust: \( T = M(V_{jet} - V_{aircraft}) \)
Thrust Power: \( \tau V_{aircraft} = M(V_{jet} - V_{aircraft})V_{aircraft} \)
Jet Power: \( P = M(V_{jet}^2 - V_{aircraft}^2) / 2 \)
Propulsion Efficiency: \( \eta_p = 2V_{aircraft}(V_{jet} + V_{aircraft}) \)

Wind Turbine

Maximum Ideal Power: \( P_{max} = 8 \rho AV_1^3 / 27 \)

Energy Equation

Pump and Turbine \( p_1/\rho g + z_1 + V_1^2/2g + w_1/\rho g = p_2/\rho g + z_2 + V_2^2/2g + w_2/\rho g \)
With Friction: \( p_1/\rho g + z_1 + V_1^2/2g = p_2/\rho g + z_2 + V_2^2/2g + h_L \)

Hydraulic Machines

Similarity Equations: \( Q_M/Q_P = (\omega_M/\omega_P) (D_M/D_P)^3 \)
\( H_M/H_P = (\omega_M/\omega_P)^2 (D_M/D_P)^2 \)
\( P_M/P_P = (\rho_M/\rho_P) (\omega_M/\omega_P)^3 (D_M/D_P)^5 \)

Pump Specific Speed: \( N_S = \omega Q^{1/2} / (gH)^{3/4} \)
Turbine Specific Speed: \( N_S = \omega P^{1/2} / [p^{1/2} (gH)^{5/4}] \)
Critical Cavitation Parameter: \( \sigma = [[(p_{atmosphere} - p_{vapour}) / \rho g] - \Delta z] / H \)
Moody Efficiency Relationship: \( \eta_P = 1 - (1 - \eta_M) (D_M/D_P)^{1/4} (H_M/H_P)^{1/10} \)
Approximate Moody Efficiency: \( (1 - \eta_M)/(1 - \eta_P) \approx (D_P/D_M)^{1/5} \)
Power: \( P = \rho g Q H \)
Pumps

Hydraulic Torque: \[ \tau = \rho Q \left( r_2 V_{2T} - r_1 V_{1T} \right) \]
Hydraulic Torque: \[ \tau = \rho Q \left( r_2 V_{2\text{cos} \alpha_2} - r_1 V_{1\text{cos} \alpha_1} \right) \]
Power: \[ P = 2\pi N \tau \]
Net Positive Suction Head: \[ \text{NPSH} = \left[ \left( \rho_{\text{atmosphere}} - \rho_{\text{vapour}} \right) / \rho g \right] - \Delta Z - h_L \]
Peripheral Velocity Factor: \[ \phi = V_{B2} / \left( 2gh \right)^{1/2} \]
Critical Cavitation Parameter: \[ \sigma_c = \text{NPSH} / H \]
Approximate Moody Efficiency: \[ (1 - \eta_p) / (1 - \eta_m) \approx \left( D_m / D_p \right)^{1/5} \]

Steam Turbines

Force on Blades: \[ F = M \left( V_{S1 \text{cos} \theta} - V_{S2 \text{cos} \delta} \right) \]
Power to Blades: \[ P = M \left( V_{S1 \text{cos} \theta} - V_{S2 \text{cos} \delta} \right) V_B \]
Power to Blades: \[ P = M \left[ \left( V_{S1}^2 - V_{S2}^2 \right) + \left( V_{R2}^2 - V_{R1}^2 \right) \right] / 2 \]