Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.

2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:

1. (a) 10 marks, (b) 10 marks
2. (a) 10 marks, (b) 10 marks
3. (a) 10 marks, (b) 5 marks, (c) 5 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks
1. Find the general solutions of the following differential equations:
   (a) \( y' + 2xy = 2xe^{-x^2} \),
   (b) \( y'' + y' - 6y = 0 \).
   Note that in each case, \( \prime \) denotes differentiation with respect to \( x \).

2. Solve the following initial value problems:
   (a) \( y' + 2ty^2 = 0, \quad y(1) = 2 \),
   (b) \( y'' - 12y' + 45y = 18 \cos(3t), \quad y(0) = 0, \quad y'(0) = 0 \).
   Note that in each case, \( \prime \) denotes differentiation with respect to \( t \).

3. Consider the two lines defined as follows:
   \( x = 3 + 2t, \quad y = 3, \quad z = 1 - t \) (parameter \( t \));
   \( x = s, \quad y = 1 - 2s, \quad z = 2 + s \) (parameter \( s \)).
   (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.
   (b) Find a third line orthogonal to both lines.
   (c) Is there a plane containing both lines? If so, find an equation for that plane.

4. Find the maximum and minimum values of \( f(x, y, z) = x + y - 2z \) over the ellipse \( x^2 + y^2 + 4z^2 = 1 \).

5. Let \( S \) be the boundary of the region enclosed by the paraboloid \( z = 4 - x^2 - y^2 \) and the plane \( z = 0 \) and let
   \[ F(x, y, z) = x^2i - 2xyj + x^2zk. \]
   Evaluate the surface integral \( \int_S F \cdot \mathbf{n} \, dA \), where \( \mathbf{n} \) is the unit outward normal on \( S \).

6. At what angle does the line represented parametrically by \( x = 1 - t, \quad y = t, \quad z = 2 + 3t \) intersect the hyperboloid \( z = 4 - x^2 + y^2 \)? You may leave your answer as an inverse sine or cosine.

7. Evaluate the line integral \( \int_C \mathbf{v} \cdot d\mathbf{r} \) where \( C \) is the curve formed by the intersection of the cylinder \( x^2 + y^2 = 4 \) and the plane \( z + 2x - y = 3 \), travelled counterclockwise as viewed from the positive \( z \)-axis, and \( \mathbf{v} \) is the vector function \( \mathbf{v} = xi + (x - y)j + yzk \).

8. Find the volume of the solid region inside the ellipsoid
   \[ x^2 + y^2 + 4z^2 = 5 \]
   and above the cone
   \[ z = \sqrt{x^2 + y^2}. \]