NOTES:

1. If doubt exists as to the proper interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement about any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.
3. This is a "Closed-Book" examination. The candidate may have a single 8.5 inch by 11 inch sheet (both sides) of hand-written notes as an aid for the examination.
4. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
5. All questions are of equal value.
6. This examination paper has 3 pages.

<table>
<thead>
<tr>
<th>Values of common constants:</th>
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<tbody>
<tr>
<td>$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m</td>
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<tr>
<td>$\mu_0 = 4\pi \times 10^{-7}$ H/m</td>
</tr>
<tr>
<td>$c = 2.998 \times 10^8$ m/s</td>
</tr>
<tr>
<td>$q = 1.602 \times 10^{-19}$ C</td>
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<tr>
<td>$h = 6.626 \times 10^{-34}$ Js</td>
</tr>
<tr>
<td>$K = 1.381 \times 10^{-23}$ J/°K</td>
</tr>
<tr>
<td>$0^\circ K = -273^\circ C$</td>
</tr>
<tr>
<td>$1 \text{ Å} = 1.0 \times 10^{-10}$ m</td>
</tr>
</tbody>
</table>

| $\varepsilon_{Si} = 11.8$ |
| $n_{Si} = 3.42$ |
| $E_{gSi} = 1.11$ eV |
| $\varepsilon_{Ge} = 16.0$ |
| $n_{Ge} = 4.01$ |
| $E_{gGe} = 0.67$ eV |
| $\varepsilon_{GaAs} = 13.2$ |
| $n_{GaAs} = 3.63$ |
| $E_{gGaAs} = 1.41$ eV |
| $\varepsilon_{InGaAsP} = 32$ |
| $r_{63} = 30$ pm/V |
| $n_0 = 2.30$ |

| LiNbO$_3$ $\varepsilon_0 = 32$ |
| LiNbO$_3$ $r_{63} = 30$ pm/V |
| LiNbO$_3$ $n_0 = 2.30$ |

Useful formulas:

\[
\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \left( \frac{x}{a} \right)
\]

\[
P(n) = \frac{N^n \exp(-N)}{n!}
\]

\[
A_{Ga_{1-x}As} \quad E_g (eV) = 1.424 + 1.266x + 0.266x^2
\]

\[
I_s = R_s \sqrt{P_o P_1 \cos \theta}
\]

\[
n(E) = n_0 - \frac{1}{2} r_3 n_0^3 E
\]

\[
x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
Question 1
(a) In an optical fiber, the phase velocity is defined as \( v_p = c/n \), while the group velocity for a narrowband signal can be defined as \( v_g = c/N \) where \( N \) is the group index.

Show that \( N = n - \lambda \frac{dn}{d\lambda} \) and derive the dispersion parameter \( \frac{d^2 \beta}{d\omega^2} \) in terms of \( N \).

(b) A step-index fiber for 1350 nm light waves has a core diameter of 20 \( \mu \)m with refractive index \( n_1 = 1.465 \) and group index \( N_1 = 1.474 \). The cladding has refractive index \( n_2 = 1.462 \) and group index \( N_2 = 1.466 \). Assume a total intramodal dispersion of 20 ps/(nm-km) and that the laser diode source has a spectral width 15 nm.

(i) Is the fiber operating single-mode or multimode? If multimode, estimate the number of modes that can propagate.

(ii) What is the approximate maximum length of this fiber for a RZ data transmission rate of 100 Mbits/s?

(c) Discuss the sources of dispersion in an optical fiber.

Question 2
A fiber optic link consists of an LED emitting light at 850 nm, multimode fiber, and a PIN photodetector with responsivity 0.65 A/W. The detector's dark current is 10 nA. The load resistor is 50 \( \Omega \). The receiver bandwidth is 50 MHz and it operates at 300 K.

(a) If the LED emits constant 5 mW of power, how much system loss can be tolerated for a SNR of 13 dB at the receiver? (Assume the thermal noise dominates).

(b) If the connector and coupling losses total 12 dB, and the fiber loss is 3.5 dB/km, what is the maximum length of the fiber link?

(c) For the situation in part(a), verify that the shot noise is much less than the thermal noise.

(d) If the LED is amplitude modulated by a sinusoidal signal with modulation index \( m = 0.5 \) and the average power emitted is still 5 mW, what is the SNR at the receiver if the system loss is 26dB?

Question 3
(a) A GaAs LED operates at 27\(^\circ\)C, with a bias current of 15 mA and it has quantum efficiency of 15%. Estimate the wavelength, the spectral width, and the intensity of the light wave from this LED.

(b) A GaAs quantum well Fabry-Perot laser has a cavity length of 200 \( \mu \)m and a lateral width of 5 \( \mu \)m. The laser emits light at a wavelength of 850 nm with a threshold current of 2 mA. The Material loss is 25 cm\(^{-1}\) and the internal quantum efficiency is 80%. The laser is operated at a current of 20 mA. Assume that the spontaneous emission can be neglected.

i) What is the threshold gain and photon lifetime in the cavity?

ii) What is the photon density per unit area in the cavity?

iii) Calculate the total light power generated inside the cavity.

iv) Calculate the light power emerging from either of the cleaved mirror facets.
Question 4
A silicon photodiode has a measured responsivity shown in the figure below. Its photosensitive area is 5mm². It is used with 30V reverse bias when the dark current is 10nA and the junction capacitance is 3pF. The transit time of the photodiode is 0.5 ns. The photodiode is used with a 50Ω load resistance and amplifier having 7pF input capacitance and operates at 300K.

(a) Explain the main features of the measured responsivity.
(b) Calculate the quantum efficiency at 850nm wavelength without reverse bias and with 30V reverse bias.
(c) What is the intensity of light at 850 nm that gives a photocurrent equal to the dark current when 30V reverse bias is applied?
(d) What is the bandwidth for detection and what is the response time?
(e) For an incident optical power of 10μW at 850nm, and 30V reverse bias, calculate the quantum noise limit and thermal noise limit of the SNR for the detector. Also calculate the NEP.
(f) Is the diode being operated in the photovoltaic mode or photoconductive mode? Sketch the I-V response curve and show the load line.
Question 5
(a) You are required to design an optical receiver which will operate at a temperature 300 K. The photodetector has a 680kΩ bias resistor. The total capacitance of the photodetector and amplifier input is 5pF. You must decide between

A: a transimpedance amplifier with $R_{in} = 1 \text{ MΩ}$, a 220 kΩ feedback resistor and open loop gain 500.

or B: an amplifier with input resistance ($R_{in}$) of 1 MΩ.

Which design would you choose based on bandwidth (without equalization) and noise performance? What is the benefit of adding an equalizer after the amplifier B?

(b) Describe the principles of operation of an erbium-doped fiber amplifier (EDFA). What are the main characteristics, advantages and disadvantages of a typical EDFA in a WDM optical system? Use sketches to illustrate your answers.

Question 6
A 1550nm single mode digital fiber optic link needs to operate at 250 Mb/s in RZ format over 75 km without repeaters. The risetime of the transmitter is 0.5ns. Excess noise penalties are predicted to be 2.5 dB. The link uses:

single mode InGaAsP laser diode: threshold current 2mA
operating current 10mA
output power 5mW
wavelength 1550nm
spectral width 5nm
RIN noise -120dB/Hz
Photon lifetime 5ps
Spontaneous recombination time 1ns

single mode fiber: input coupling loss 1.0dB
output coupling loss 0.5dB
splice loss 0.2 dB every km
fiber loss 0.25dB/km
dispersion 2.5 ps/(nm-km)

photodetector: InGaAs PIN
sensitivity -36dBm
responsivity 0.6A/W

(a) Set up an optical power budget for this link and find the system margin. Determine the detector photocurrent.

(b) What is the maximum system risetime for the link? Choose an appropriate receiver bandwidth (explain your choice) and show that the system risetime requirement is met.

(c) Would your design work if the data was in NRZ format instead of RZ format? Explain your answer.

(d) What is the maximum data rate that the laser diode could be directly modulated? What problems can occur with direct modulation?