04-CHEM-B1, TRANSPORT PHENOMENA

May 2016

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. The examination is an open book exam. One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.

3. Candidates may use any non-communicating calculator.

4. All problems are worth 25 points. One problem from each of sections A, B, and C must be attempted. A fourth problem from any section must also be attempted.

5. Only the first four questions as they appear in the answer book will be marked.

6. State all assumptions clearly.
Section A: Fluid Mechanics

A1. The simplest patient infusion system is that of gravity flow from an intravenous (IV) bag. A 500 ml IV bag containing an aqueous solution is connected to a vein in the forearm of a patient. Venous pressure in the forearm is 0 mm Hg (gage pressure). The IV bag is placed on a stand such that the entrance to the tube leaving the IV bag is exactly 1 meter above the vein into which the IV fluid enters. The length of the IV bag is 30 cm. The IV is fed through an 18-gage tube (internal diameter = 0.953 mm) and the total length of the tube is 2 meters. Calculate the flow rate of the IV fluid, and estimate the time needed to empty the IV bag.

A2. The spin coating process is used to coat a thin film of photoresist (organic polymer) on the wafer. A viscous solution containing the photoresist material is made to form a thin (~ 1 µm) uniform thin film on the wafer by spinning the wafer as shown in the figure below.

![Diagram of spin coating process]

a) [22 points] Derive an expression for thickness of the film on the wafer (h) as a function time (t), angular velocity (ω), and the kinematic viscosity (ν) of the photoresist solution.

b) [3 points] What is the corresponding equation for thickness of a very thin film of photoresist on the wafer?
Section B: Heat Transfer

B1. Air at 1 atm and 150 °C enters a pipe of diameter 5.08 cm, and it is heated as it moves through the pipe at a velocity of 8 m/s. Assume constant heat flux per unit length of the pipe, and that the pipe wall temperature is 20 °C above air temperature.

a) [18 points] Determine the heat transfer per unit length of the pipe.

b) [7 points] Calculate the bulk temperature rise through a 2-meter length of pipe.

DATA:

Thermal conductivity of air = 3.52 x 10^{-2} W/m °C
Viscosity of air = 2.38 x 10^{-5} kg/m.s
Specific heat capacity of air = 1.017 kJ/kg °C
Molecular weight of air = 29 g/mol

B2. Derive the differential energy balance equation in spherical coordinates for heating a sphere of radius r, if the temperature variation with angular position is zero.
Section C: Mass Transfer

C1. For rate of mass transfer from a cylinder rotating in a liquid bath, the Sherwood number (Sh) is given by the equation

\[ Sh = \frac{k_d}{U_r} = 0.079 \left( \frac{Re}{Sc} \right)^{0.3} \left( \frac{Sc}{Re} \right)^{-0.64} \]

where \( k_d \) is the mass transfer coefficient between cylinder and liquid, \( U_r \) is the peripheral speed of rotation, Schmidt number \( (Sc) = \frac{\nu}{D_{AB}} \), Reynolds number \( (Re) = \frac{dU}{\nu} \), and kinematic viscosity \( (\nu) = \frac{\nu}{\rho} \).

In an experiment, a 1.5-cm diameter by 5.0-cm long iron cylinder was rotated at a speed of 77 rpm in a melt of carbon-saturated iron at 1275 °C. The rate of dissolution, expressed as decrease of rod radius with time, was \(-5.57 \times 10^{-3}\) cm/s. For the dissolution of iron in carbon-saturated iron, the iron mass concentration driving force \( (AX_{Fe}) \) was estimated to be 0.385. Assuming the densities of the iron rod and the carbon-saturated iron melt are equal, compare the experimental rate of dissolution with that predicted by the above correlation.

DATA:

Diffusivity of iron in the carbon-saturated iron melt at 1275 °C = \(9 \times 10^{-5}\) cm²/s
Kinematic viscosity of iron melt at 1275 °C = \(1.41 \times 10^{-2}\) cm²/s

C2. During class break, a guy comes and cleans the blackboards. He leaves a thin film of water whose thickness is 1 mm. Water evaporates from that surface into the surrounding air, whose humidity is 10%. We want to calculate how long it takes for the water to evaporate completely. Assume a diffusional distance between the water surface and ambient air of 1 cm. At 25 °C, the vapor pressure of water is 0.0313 atm, and the diffusion coefficient of water vapor in air is 0.282 cm²/s.

a) [3 points] Define the problem, draw coordinates, and state assumptions.

b) [13 points] Calculate the steady-state flux of water from the surface.

c) [2 points] Obtain an expression to relate the flux to the disappearance of liquid water.

d) [7 points] Calculate the time required for the water film to evaporate and the chalkboard to dry completely.
APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

<table>
<thead>
<tr>
<th>Coordinate System</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular coordinates (x, y, z)</td>
<td>[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 ]</td>
</tr>
<tr>
<td>Cylindrical coordinates (r, ( \theta ), z)</td>
<td>[ \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho u_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho u_\theta) + \frac{\partial}{\partial z} (\rho u_z) = 0 ]</td>
</tr>
<tr>
<td>Spherical coordinates (r, ( \theta ), ( \phi ))</td>
<td>[ \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho r^2 u_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho u_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (\rho u_\phi) = 0 ]</td>
</tr>
</tbody>
</table>

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant \( \rho \) and \( \mu \)

<table>
<thead>
<tr>
<th>Coordinate System</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular coordinates (x, y, z)</td>
<td>[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla P + \mathbf{\tau} + \nu \nabla^2 \mathbf{u} ]</td>
</tr>
<tr>
<td>x-component</td>
<td>[ \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + g_x + \nu \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) ]</td>
</tr>
<tr>
<td>y-component</td>
<td>[ \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + g_y + \nu \left( \frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) ]</td>
</tr>
<tr>
<td>z-component</td>
<td>[ \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \left( \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) ]</td>
</tr>
</tbody>
</table>
Cylindrical coordinates $(r, \theta, z)$

\[
\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} = \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial u_r}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - 2 \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} + \frac{\partial^2 u_r}{\partial \beta^2}
\]  

(A2d)

\[
\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} = -\frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial u_\theta}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + 2 \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{\partial^2 u_\theta}{\partial \beta^2}
\]  

(A2e)

\[
\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial u_z}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} + \frac{\partial^2 u_z}{\partial \beta^2}
\]  

(A2f)

Spherical coordinates $(r, \theta, \phi)$

\[
\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{\partial}{\partial \theta} \left( \frac{u_\theta}{r \sin \theta} \frac{\partial u_r}{\partial \theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial u_r}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \left( \frac{u_\phi}{r \sin \theta} \frac{\partial u_r}{\partial \phi} \right) = \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial \rho}{\partial \theta} \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left( \frac{\partial \rho}{\partial \phi} \right) + \frac{1}{\rho} \frac{\partial}{\partial \phi} \left( \rho \frac{\partial \rho}{\partial \phi} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \left( \frac{\partial \rho}{\partial \phi} \right)
\]  

(A2g)

\[
\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi} + \frac{\partial}{\partial \theta} \left( \frac{u_\theta}{r \sin \theta} \frac{\partial u_\theta}{\partial \theta} \right) + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial u_\theta}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2 \cot \theta}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi} = \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial \rho}{\partial \theta} \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left( \frac{\partial \rho}{\partial \phi} \right) + \frac{1}{\rho} \frac{\partial}{\partial \phi} \left( \rho \frac{\partial \rho}{\partial \phi} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \left( \frac{\partial \rho}{\partial \phi} \right)
\]  

(A2h)

\[
\frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{\partial}{\partial \theta} \left( \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \theta} \right) + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial u_\phi}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 u_\phi}{\partial \theta^2} + \frac{2 \cot \theta}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} = \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho \frac{\partial \rho}{\partial \theta} \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left( \frac{\partial \rho}{\partial \phi} \right) + \frac{1}{\rho} \frac{\partial}{\partial \phi} \left( \rho \frac{\partial \rho}{\partial \phi} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \left( \frac{\partial \rho}{\partial \phi} \right)
\]  

(A2i)
Table A.3 The Energy Equation for Incompressible Media

\[ \rho c_p \left( \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla)T \right) = \nabla \cdot (k \nabla T) + \dot{Q}_G \]  

(A3)

**Rectangular coordinates \((x, y, z)\)**

\[ \rho c_p \left[ \frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \right] = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{Q}_G \]  

(A3a)

**Cylindrical coordinates \((r, \theta, z)\)**

\[ \rho c_p \left[ \frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} \right] = \frac{1}{r} \frac{\partial}{\partial r} \left( r k \frac{\partial T}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( k \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \dot{Q}_G \]  

(A3b)

**Spherical coordinates \((r, \theta, \phi)\)**

\[ \rho c_p \left[ \frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right] = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 k \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \dot{Q}_G \]  

(A3c)

---

Table A4: The continuity equation for species \(A\) in terms of the molar flux

\[ \frac{\partial C_A}{\partial t} = -\left( \nabla \cdot \mathbf{\bar{N}}_A \right) + \dot{K}_{A,G} \]  

(4)

**Rectangular coordinates \((x, y, z)\)**

\[ \frac{\partial C_A}{\partial t} = -\left( \frac{\delta [N_A]}{\delta x} + \frac{\partial [N_A]}{\partial y} + \frac{\partial [N_A]}{\partial z} \right) + \dot{K}_{A,G} \]  

(4a)

**Cylindrical coordinates \((r, \theta, z)\)**

\[ \frac{\partial C_A}{\partial t} = \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left[ r N_A \right] + \frac{1}{r} \frac{\partial}{\partial \theta} \left[ N_A \right] + \frac{\partial}{\partial z} \left[ N_A \right] \right\} + \dot{K}_{A,G} \]  

(4b)

**Spherical coordinates \((r, \theta, \phi)\)**

\[ \frac{\partial C_A}{\partial t} = \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 N_A \right] + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left[ N_A \sin \theta \right] + \frac{1}{r \sin \theta \sin \phi} \frac{\partial}{\partial \phi} \left[ N_A \sin \phi \right] \right\} + \dot{K}_{A,G} \]  

(4c)
Table A.5: The continuity equation for species $A$

$$\frac{\partial C_A}{\partial t} + (\vec{u} \cdot \nabla) C_A = D_A \nabla^2 C_A + \dot{R}_{A,G}$$  \hspace{1cm} (5)$$

**Rectangular coordinates $(x, y, z)$**

$$\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x} \left( D \frac{\partial C_A}{\partial x} \right) + \frac{\partial}{\partial y} \left( D \frac{\partial C_A}{\partial y} \right) + \frac{\partial}{\partial z} \left( D \frac{\partial C_A}{\partial z} \right) + \dot{R}_{A,G}$$  \hspace{1cm} (5a)$$

**Cylindrical coordinates $(r, \theta, z)$**

$$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r D \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( D \frac{\partial C_A}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( D \frac{\partial C_A}{\partial z} \right) + \dot{R}_{A,G}$$  \hspace{1cm} (5b)$$

**Spherical coordinates $(r, \theta, \phi)$**

$$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r^2 \sin \theta} \frac{\partial C_A}{\partial \phi} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 D \frac{\partial C_A}{\partial r} \right)$$

$$+ \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial C_A}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( D \frac{\partial C_A}{\partial \phi} \right) + \dot{R}_{A,G}$$  \hspace{1cm} (5c)$$